PROFIT MAXIMIZATION IN THE NATIONAL FOOTBALL LEAGUE (NFL)

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John Patrick Brunkhorst
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PROFIT MAXIMIZATION IN THE NATIONAL FOOTBALL LEAGUE (NFL)

John Patrick Brunkhorst

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Abstract

This paper investigates the hypothesis of the profit maximization theory as it applies to the National Football League (NFL). A profit maximization function is constructed incorporating variable revenue and cost factors such as gate receipts and player expenses. A systems model is used as the estimation procedure to identify the determinants of ticket prices for NFL franchises. This model implies parameter restrictions across two equations to incorporate the Kuhn-Tucker conditions. Results from the regression are then used in conjunction with other data to numerically test the derived profit maximization conditions. The results support profit maximizing behavior by NFL teams, indicating that over 80% of teams set ticket prices at a level corresponding to profit maximization.

KEYWORDS: (National Football League, Profit Maximization, Ticket Price Determination)
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# TABLE OF CONTENTS

## I  INTRODUCTION  

## II  LITERATURE REVIEW  

Economic Overview................................................................. 7  
Classical Theories..................................................................... 11  
Recent Empirical Studies....................................................... 14  
Profit Maximization in Sports............................................... 18  
  Individual Sports.............................................................. 19  
  Team Sports......................................................................... 20  
  Uncertainty.......................................................................... 22  
Entertainment........................................................................... 23  
Territory.................................................................................... 24  
Revenue Sharing....................................................................... 25  
Incentives.................................................................................. 27  
Empirical Research................................................................. 27

## III  THEORY  

Profit Maximization in the NFL............................................... 34  
The Profit Maximization Problem.............................................. 36  
Theoretical Determinants of Profit Maximization in the NFL.... 41  
  Territory................................................................................ 42  
  Uncertainty.......................................................................... 44  
  Star Power.......................................................................... 47  
Conclusion................................................................................ 48

## IV  DATA AND METHODS  

Dependent Variable............................................................... 52  
Other Endogenous Variables.................................................. 53
APPENDIX C................................................................................................. 93

APPENDIX D................................................................................................... 101

SOURCES CONSULTED................................................................................... 103
LIST OF FIGURES

1.1 Leeds and Von Allmen’s Profit Model ............................................ 2
2.1 Literature Pertaining to Profit Maximization in the NFL................. 7
2.2 Profit Maximization: TR – TC Approach....................................... 9
2.3 Profit Maximization: MR – MC Approach.................................. 10
2.4 Equation Stating Variables used in Boyd and Boyd’s (2001) Study.... 31
3.1 Factors Affecting Profit Maximization in the NFL......................... 42
4.1 Simultaneous Empirical Model for Ticket Prices in the NFL ............ 51
4.2 Equation for Herfindahl Hirschman Index................................. 57
5.1 First and Second-Order Derivatives for Profit with Respect to Attendance 74
5.2 Conditions of Profit Maximization ........................................ 77
LIST OF TABLES

2.1 Variables in Tauer's (1995) Study ................................................. 16
2.2 Statistics used in Levitt's (2006) Study ........................................... 17
2.3 Variable Definitions for Ferguson et al. (1991) Study ......................... 29
2.4 Variable Definition for Boyd and Boyd's (2001) Study .......................... 31
4.1 Variable Definitions .............................................................................. 51
5.1 The Determinants of Ticket Price ......................................................... 65
5.2 Numerical Results for the First and Second Derivatives of the Profit Function 75
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CHAPTER I
INTRODUCTION

The theory of profit maximization has been a key economic concept for almost a century.\(^1\) Profit maximization occurs when firms adjust either the quantity or price of the goods they produce in order to return the largest profit. This idea has been considered one of the primary goals for almost every industrial business. Professional sports leagues have been one of the leading markets in the entertainment industry for a length of time. So why would sports leagues like the National Football League (NFL) be an exception to this theory?

The NFL is an economic enterprise where every single team in the league is consistently profitable.\(^2\) From salary caps to ticket prices, and multimillion-dollar contracts to Super Bowl television ads, issues concerning money are always seen surrounding the league and its teams. Since all NFL franchises are businesses it would only make sense that the core business practices would hold true in their case. Profit maximization would presumably be one of these practices. In spite of this, neither profit maximization nor utility maximization has yet proven to be the dominant behavior of NFL owners. Zimbalist (2003) states that individual owner’s economic objectives differ

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\(^1\) Frank Hyneman Knight, "Risk, Uncertainty and Profit," (1921): 9

due to the unique way each owner defines profit maximization. Some owners focus simply on money, while others believe the best way to profit maximize is to win maximize. Utility maximization, applied to sports, is the theory that all teams try to win the greatest number of games possible. This theory applies to basically all teams up to the professional level. However, when money is brought into the picture, the dynamics of sports change. Now owners and teams are not only satisfied with winning, but also desire to make money.

When examining profit maximization in a professional sport market, Leeds and Von Allmen’s profit model is the most common approach used. In this model, profit is presented as the difference between total revenue and total cost. FIGURE 1.1 outlines Leeds and Von Allmen’s model.

FIGURE 1.1
Leeds and Von Allmen’s Profit Model

\[ \pi = TR - TC \]

\[ TR = R_g + R_b + R_l + R_s \]

\[ TC = C_p + C_t + C_m + C_o + C_v \]

In this model, \( \pi \) represents profit and \( TR \) represent total revenue, which is the sum of gate receipts (\( R_g \)), broadcasting rights (\( R_b \)), licensing sales (\( R_l \)), and other stadium related income (\( R_s \)). In the NFL, national broadcasting rights and licensed merchandise sales

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5 Ibid.
are equally divided among teams. Stadium revenues are also assumed to be fairly consistent across teams. Therefore, these three types of revenue are assumed to be fixed, leaving gate receipts as the only form of variable revenue between teams. TC represents total cost, which is comprised of player expenses ($C_p$), travel ($C_d$), marketing ($C_m$), administration ($C_a$), and venue expenses ($C_v$). Player expenses usually account for the majority of total cost, while the other forms of costs are small and fairly equivalent across teams. Therefore, player expenses is the primary variable cost of NFL franchises.

Combining these views on revenue and cost, a profit maximization study analyzing NFL franchises should focus on the difference between gate receipts and player expenses as a measure of profit: $\pi = R_g - C_p$. Only when an owner maximizes this difference between gate receipts and player expenses will the franchise uphold the theory of profit maximization.

However, as mentioned earlier, all owners don’t have the same thoughts on profit. Even though research conducted by Forbes shows that each team in the NFL makes significant profit, that does not mean each team maximizes potential profits. Constant sellouts and scalpers selling tickets above face value point to the fact that NFL franchises could raise ticket prices in order to increase profits. Perhaps some owners are more interested in winning championships than making the most money. As long as franchises are not losing money, their ultimate goal could be the glory and prestige accompanying a Super Bowl victory. On the other hand, constantly increasing ticket prices and television

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blackouts in local areas during non-sellout games all point to making profit as the main objective. In light of these observations, the question is posed: do NFL franchises conform to the practices of profit maximization?

When examining this question numerous factors change the potential profit each NFL franchise can make. Variables completely outside the league such as the wealth, population, and alternative forms of entertainment in a franchise's market area differ from team to team. Inside the league, the current and previous success of franchises, competitive balance between teams, and the players on each team affect the number of fans at a game. Previous research by Boyd and Boyd (2001) and Ferguson et al. (1991) have studied the effects of these variables on ticket prices in other sports leagues. Therefore each variable should play a role in determining ticket prices for an NFL football game. It also needs to be mentioned that the supply of NFL tickets is limited. There is a ceiling placed on the number of tickets sold for an NFL game due to the capacity restriction each stadium carries with it. This restriction needs to be incorporated because it not only affects ticket prices, but also the profit maximizing behavior. Little research has been conducted focusing on profit maximization in professional sports leagues, with none of it addressing the NFL.

The goal of this study is to determine whether franchises and owners in the NFL make decisions regarding ticket prices in a manner consistent with the profit maximizing conditions. The following chapter will provide a discussion of the pertinent literature concerning profit maximization, including its history, place in the current economics, and

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relationship with professional sports. Chapter three will focus on the theories behind the profit maximizing behavior of NFL owners. A mathematical model for both profit and ticket prices will be outlined in detail. These theoretical models developed in this chapter will serve as the foundation for testing the hypothesis of profit maximization in the NFL. The profit maximization conditions and numerical equations used for testing will also be addressed in this chapter. The fourth chapter will present the data set and empirical model used for determining NFL ticket prices in this study. It will also discuss each variable individually and the effects it has on the model. Finally, chapter five will provide all the results for both the empirical model and the numerical testing of profit maximization. Any conclusions drawn from these results will be presented, and lastly suggestions for possible future research and implications will be addressed.
CHAPTER II

LITERATURE REVIEW

This chapter is devoted to discussing the past and current research pertaining to this study of profit-maximization in the National Football League (NFL). Over the last half century the NFL has become more of a business than ever. Everywhere one looks in the media there is always a story concerning salary caps, teams folding, or increasing ticket prices. At the professional level, football is not longer just a game to determine which team is the best. Owners care more about money and less about winning. Previous studies have focused on profit maximization versus utility maximization in trying to determine if teams play for money or competitiveness. It is essential to look at profit maximization in the NFL from the top down. One must understand the logical flow of profit maximization from classical theory to modern research in both the industrial and sports world. FIGURE 2.1 provides an overview of literature pertaining to this study of profit maximization in the NFL.
Economic Overview

Economic theory is based on the reasonable assumption that people are motivated to do as best as they can for themselves given the situations and constraints facing them.¹ Every person has a different idea of satisfaction. Some people enjoy leisure while others enjoy working. In the business world, owners attempt to manage their business in order to improve their own well-being. They seek satisfaction through earning as much profit as possible. The best firms use creativity and excellence to become most efficient firm in converting scare resources into goods and services.²

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the efficiency of good business practice and ethics. A firm who chooses both inputs and outputs with the sole goal of achieving maximum economic profit is said to be profit maximizing. In many industries profit maximization is not just a goal, it is the only goal. By earning the greatest profit businesses can drive their competitors out of the market, allowing them to increase their own profits even more.

A firm maximizes profit when it desires to earn the most possible and adjusts operations to raise profit. To do this, a firm determines the price and quantity of output that will return the largest profit. Two approaches can be used to understand profit maximization. The first method examines total revenue (TR) and total cost (TC). This method relies on the definition that profit equals revenue minus cost. Revenue is the total amount of money that flows into a firm, which in most models is price times the quantity of goods sold. Total costs can be divided into two groups: fixed and variable. Fixed costs are the same for the company no matter what their level of output. Variable costs are any costs that change depending on the level of output. These usually increase as output level increases. In the TR-TC approach it is quite simple to determine the level of output where profit maximization occurs. Profit maximization occurs at the point where TC-TR is the greatest. FIGURE 2.2 illustrates a graph of profit maximization using the TC-TR approach. The figure shows the TR curve, the TC curve, and also the Profit curve.

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3 Ibid., 18
7 Primeaux and Stieber, "Profit Maximization: The Ethical Mandate of Business," 21
The largest difference between TR and TC occurs at the quantity $Q$. Also at quantity $Q$, the point $B$ on the TC curve will have a tangent that is parallel to the TR curve.

FIGURE 2.2

Profit Maximization: TR – TC Approach

The second approach to profit maximization is marginal revenue (MR) – marginal cost (MC). Marginal revenue and cost are the change in revenue that occurs if one additional unit of output is produced. It can also be looked at as the derivative of revenue or cost when the level of output is a function with respect to either.\(^8\) Therefore marginal profit equals MR – MC. When MR is greater than MC, marginal profit is positive, meaning the total profit is increasing. When MR is less than MC, marginal profit is negative, meaning total profit is decreasing. When MR does not equal MC the firm is in

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a state of disequilibrium.\textsuperscript{9} Total profit can only reach a maximum when marginal profit equals zero. Only when MR=MC can a company be in equilibrium creating maximum profit.\textsuperscript{10} Therefore profit maximization occurs at a point when MR – MC = 0. A basic example of the MR–MC approach can be seen in FIGURE 2.3. In this figure MR stays the same for each additional unit of output. MC curves down then back up until it intersects MR at quantity Q. Therefore Q represents the quantity where profit maximization occurs.

FIGURE 2.3
Profit Maximization: MR–MC Approach

Although the first order derivative shows us at the profit maximizing quantity MR must equal MC, it is not the only condition necessary. Nicholson (1985) states it is also necessary to find the second order derivative at that quantity.\textsuperscript{11} A profit curve, which can be seen in FIGURE 2.2, can have both a maximum and a minimum. At both the


\textsuperscript{10} Ibid.

\textsuperscript{11} Nicholson, "Microeconomic Theory : Basic Principles and Extensions," 366-367
minimum and maximum profit levels the first order derivative will show MR=MC. By looking at the second order derivative one can tell if profit has an increasing or decreasing rate at the quantity where MR=MC. If q is a profit maximizing quantity, then the second order derivative at q is negative. If q* is a profit minimizing condition then the second order derivative will be positive. If and only if the first and second order derivatives at a certain quantity fulfill the requirements can that quantity be the actual profit maximizing quantity of a firm.

Classical Theories

The profit maximization theory is by no means a recent discovery to the field of economics. The idea has been found in literature since the beginning of the 20th century. Knight (1921) describes economics as the study of a particular form of human want-satisfying activity that has become prevalent in Western Nations such as the U.S.12 The static approach to economics examines the existing current conditions and the results known forces tend to produce under those conditions. A competitive market is the primary model used. Under perfect conditions, the competitive market model will produce equality, where production equals costs, thus eliminating profit. However, in actual society, production and cost only “tend” towards equality. Rarely are they precisely equal. The difference between production and cost is the margin of “profit,” which can be negative or positive. Knight (1921) states the existence of uncertainty is the

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12 Frank Hyneman Knight, "Risk, Uncertainty and Profit," (1921): 9
basis for the theory of profit and creates the divergence between actual and theoretical competition.¹³

Profit in its simplest definition is the difference between revenue and costs. If a company has greater revenue than cost it earns a profit. If revenue is less than cost it is said to suffer a loss. Von Mises (1951) states that profits are a result of entrepreneurs judging the future market correctly.¹⁴ Entrepreneurs take a risk that involves some degree of uncertainty it will fail. Knight (1921) says profits are the rewards of risk-taking.¹⁵ These entrepreneurs buy certain factors of production at prices that seem too low at the time. They are able to use these cheaper goods in the future to lower the costs of production. They will continue to make a temporary profit until consumer demand adjusts to the prices of production. Profits are constantly in a cycle from high to low because of ceaseless changes in the economy and new adjustments being made. Those entrepreneurs who use their knowledge and skills to earn the most profit possible are considered to operate profit maximizing firms.

Carlson (1956) presents main ideas on profit maximization that are still seen in current literature.¹⁶ He identifies net return difference between total revenue and minimum costs. Total revenue is all revenue received from both outputs and investments. Minimum costs are composed of the costs of production and interests costs from borrowed funds. Rate of return is then found by expressing net returns as a rate of the

¹³ Ibid., 16


¹⁵ Knight, "Risk, Uncertainty and Profit," 25

firm’s funds over a period of time. The maximization of this rate is the firm’s goal.
Carlson (1956) also outlines the profit maximization theory using “marginal” terms.\(^{17}\) He
says the profit maximizing quantity is only reached when first order derivative shows
marginal revenue and costs to be equal, and the second order derivative is negative.
Changes in production will only be made if the change is expected to produce a greater
increase in revenue than costs.

Finally, Carlson (1956) describes the four areas where changes can occur in order
to profit maximize.\(^{18}\) They can occur in the technical process of production. By
increasing the technical efficiency in an area of production both total and marginal costs
will decrease. Firms can also change the supply conditions of production. This change
could lower either fixed or variable costs, leading to an increase in profits. A change in
supply of capital funds would have the same effects as changing the supply conditions of
production. Lastly, a change in the demand of a firm’s output would influence total and
marginal revenue, causing shifts in the volume of production and rate of return. All four
of these options are still used by firms in today’s economy.

Even though profit maximization is firmly grounded in classic economic theory it
has constantly been under attack. The main argument against it is on the grounds that it
lacks realism. Koplin (1963) states that many people believe the theory fails to account
for the alternative motives of businessmen such as power, prestige, and other non-
monetary rewards.\(^{19}\) Koplin’s (1963) paper clarifies that the profit maximization theory

\(^{17}\) Ibid., 62

\(^{18}\) Ibid., 67-73

is not always consistent with price theory, but that it is consistent with efficiency. In his model, profit maximization simply means the organization of relationships within a firm as to maximize the residual gain or loss that occurs to owners. Gains and losses can be in both monetary and non-monetary forms. An owner will decline a policy bringing him more income if the extra income does not compensate him for the added effort. Owners strive for efficiency. A failure of the firm to maximize profits can only be a result of economic inefficiency. In the end, Koplin (1963) comes to the conclusion that all owners try to profit maximize through efficiency. His paper shows profit maximization theory certainly has flaws because it only deals with monetary values, but it in no way rejects the theory. All owners do profit maximizes; the problem arises due to variations in owners’ views on what exactly “profit” entails.

Recent Empirical Studies

Current case studies in the agricultural sector of profit maximization have produced results applicable to other industries. A study by Tauer (1995) tries to determine how strictly New York dairy farmers adhere to profit maximization and cost minimization. The study states most economists would accept profit maximization or cost minimization as the sole objective of farmers. A few others argue farmers do not have perfect information, leaving them unable to make rational decisions towards profit maximization. Forty years ago a survey done by the Interstate Managerial Survey

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20 Ibid.
21 Ibid.
concluded that 95% of farmers behaved in manners consistent with static profit maximization behavior when input and output prices change.\textsuperscript{23} However, the survey further concluded many farmers' adjustments were due to disequilibrium, which violates profit maximization. Tauer (1995) states even if farmers are unable to profit maximize due to a predetermined output, they are still able to minimize their costs.\textsuperscript{24} To conduct his test, Tauer (1995) uses the Weak Axiom of Profit Maximization (WAPM) and Weak Axiom of Cost Minimization (WACM) to analyze failures in production. Failures to profit maximize are due to a combination of technical and allocative inefficiency on the part of farmers.

Tauer's (1995) study looked at data obtained from 49 New York Dairy farms from 1977 through 1987.\textsuperscript{25} Variables are obtained from the New York Dairy Farm Business Summary. The variables and their definitions are seen in TABLE 2.1

\textsuperscript{23} Ibid.
\textsuperscript{24} Ibid.
\textsuperscript{25} Ibid.
TABLE 2.1
Variables in Tauer’s (1995) Study

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feed</td>
<td>Cost of purchased feed</td>
</tr>
<tr>
<td>Animals</td>
<td>Costs of livestock and their expenses</td>
</tr>
<tr>
<td>Fuel</td>
<td>Costs of fuel and electricity</td>
</tr>
<tr>
<td>Fertilizer</td>
<td>Costs of fertilizer</td>
</tr>
<tr>
<td>Seed</td>
<td>Costs of machinery</td>
</tr>
<tr>
<td>Buildings</td>
<td>Costs of buildings and fences</td>
</tr>
<tr>
<td>Services</td>
<td>Costs of farm services such as breeding,</td>
</tr>
<tr>
<td></td>
<td>veterinarian, etc. and rent</td>
</tr>
<tr>
<td>Chemicals</td>
<td>Costs of spraying and other expenses</td>
</tr>
<tr>
<td>Wages</td>
<td>Costs of labor</td>
</tr>
<tr>
<td>Taxes</td>
<td>Costs of property tax</td>
</tr>
<tr>
<td>Milk</td>
<td>Sales of milk, dairy, livestock, etc.</td>
</tr>
</tbody>
</table>

Tauer’s (1995) WAPM results report a 49.5% average failure rate for profit maximization in New York dairy farms.\textsuperscript{26} However, the results do state that technological change and learning are being exhibited in the fact that firms appear to be more profitable in each succeeding year. When testing cost minimization, the WACM results show only a 31% failure rate, with almost all violations being minor. On average, more than 50% of farms choose inputs that are within 10% of minimum costs. Since cost minimization is a subset of profit maximization it makes sense that violations should be less. In the end, Tauer’s (1995) the results suggest profit maximization is not supported by dairy farmers, but cost minimization is strongly supported.\textsuperscript{27}

\textsuperscript{26} Ibid.

\textsuperscript{27} Ibid.
Another study by Levitt (2006) analyzes the extent to which the behavior of a bagel and donut business makes profit maximizing decisions.\textsuperscript{28} He states all models of production start with the fundamental and widely accepted theory of profit maximization. However, attempts to empirically test this assumption are rare due to the fact that firms are quite complex, producing multiple goods with many inputs.\textsuperscript{29} Levitt (2006) believes the owner of the bagel business, who is well trained in the principles of profit maximization from his ABD in Economics at MIT, should adhere to the concepts of profit maximization.\textsuperscript{30} To conduct the experiment Levitt (2006) was given company data from 1993 to 2005. The variables used can be seen in TABLE 2.2.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
Statistics & \\
\hline
Number of bagels delivered & \\
Number of bagels eaten & \\
Number of donuts delivered & \\
Number of donuts eaten & \\
Posted price of bagel (nominal $) & \\
Posted price of donuts (nominal $) & \\
Payment rate & \\
Marginal cost of bagel (nominal $) & \\
Marginal cost of donut (nominal $) & \\
Year & \\
\hline
\end{tabular}
\caption{Statistics used in Levitt’s (2006) Study}
\end{table}

Levitt’s (2006) tests find that on any given day, at a set price, the business exhibits substantial skill in changing its behavior to deliver the optimal amount of both


\textsuperscript{29} Ibid.

\textsuperscript{30} Ibid.
In 80,731 deliveries, 92.2% of all bagels and donuts are consumed. The expected profit for the last bagel delivered is .008 dollars. The expected profit from the last donut delivered is .021 dollars. Both stats show the firm’s last unit of output has a marginal profit of approximately 0, satisfying profit maximizing behavior. However, examination of profit maximization under the firm’s pricing decisions tells a different story. Levitt’s (2006) study states the firm sacrifices an average 30% loss of potential profit due to mispricing. This equates to roughly $25,000 each year. This study concludes that firms in general usually do an exceptionally good job making short-run decisions in output. However, even with a sophisticated owner, firms oftentimes deviate from optimal pricing levels in the long-run due to lack of feedback and research in pricing.

Profit Maximization in Sports

For a long time professional sporting contests have been one of the most significant branches of the entertainment industry. Szymanski (2003) reports that in 1997 the U.S. Census Bureau found 41% of the population attends a spectator sporting event each year (roughly 110 million people total). Szymanski (2003) also states that Kagan Media estimated the annual household television viewing of sports events to be 77 billion hours per year. With such a huge market its not surprising the world of professional

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31 Ibid.
32 Ibid.
34 Ibid.
sports has a large place in economics. The area of sports economics examines both individual and team sports. Each has its own unique models and theories, and it is important to understand the main differences between the two.

**Individual Sports**

In individualistic sports, such as tennis and golf, players themselves choose when and where they participate. They typically enter a competition to determine who is the best, which is also what interests spectators. Szymanski (2003) says players agree to specific rules set out by the tournament in order to compete for the winnings, which are usually associated with money and status. Players have little long-term commitment to any tournament, and can select which events will maximize their own well-being. The inverse is also true since tournaments make little commitments to players and only offer invitations to those who they deem the best or most desirable.\(^{35}\) Players are only paid on their performance in a competition. The separation of tournament and player makes individual sports conform to a standard contest model.

In individual sporting events the organizer's objective is to design a competition that maximizes efforts of those who participate. Szymanski (2003) says that spectators are drawn to contests by quality of the field and effort put forth by athletes.\(^{36}\) Athletes who participate are trying to win a prize. Prizes can be allocated in two distinct ways: winner-take-all or multiple. In a winner-take-all situation the athlete who wins the tournament collects the only prize. In multiple prize contests prizes are award to more

\(^{35}\) Ibid.

\(^{36}\) Ibid.
than one participant depending on his or her results in the contest. In both scenarios Szymanski (2003) finds individual effort and aggregate effort increase with the value of the prize. Larger fields usually decrease the amount of individual effort. Therefore in contests of high prestige, which draw many contestants, it is essential for the organizers to limit the field in some way to encourage more effort. With a large prize and a limited field, organizers are almost guaranteed to have a successful contest.

Team Sports

Professional team sports in the U.S., like basketball and football, have a unique economic structure. U.S. professional team sport leagues share common regulations like fixed number of teams, entry by only expansion, exclusive territories, et al. These rules allow leagues to act more like monopolies or cartels. Fort and Quirk state, “Professional team sport leagues are classic, even textbook, examples of business cartels.” Sports leagues are the sole provider of the product, allowing the league and team owners to set prices according to the total market demand. Therefore, Sandy, Sloane, and Rosentraub (2004) state the motives of sports team owners need to be taken into account because monopolies have more power in deciding pricing and output levels than competitive markets.

In U.S. team sport leagues owners are in sole control of the team. Owner’s motives are oftentimes quite confusing. Zimbalist (2003) states there are many reasons to

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37 Ibid.


own a sport team such as fun, power, ego, investment, developing new business relationships, political influence, and of course profit. Even within leagues the motives of different owners might vary.\textsuperscript{40} The underlying fact, however, is pro sports teams are businesses and all business owners are assumed to maximize profits. This is emphasized by the fact that there is little evidence that owners have ever received less than the market rate of return on their investment.\textsuperscript{41} Under the profit maximizing hypothesis owners make decisions on players, ticket prices, and media contracts in order to maximize the difference between total revenue and cost.\textsuperscript{42} Profits may not be great but what matters is profits are pursued. Even when teams win the championship owners might not be happy. This was shown in two occasions in Major League Baseball (MLB). Charlie Finley’s Oakland Athletics in the late 1970’s and Wayne Huizinga’a Miami Marlins in late 1990’s were World Series winning baseball clubs. Both owners realized their respective cities cared so little about baseball that even in a year when they won the championship they could not make a profit.\textsuperscript{43} Therefore both owners tried to sell all of the good players on the team in order to increase profits by lowering costs. This serves as a prime example of owners caring more about profit and little about utility.

In professional U.S. team sports the team is always considered more important than the individual. Players might receive publicity and fame but owners primarily focus on the team. In general the player’s only responsibility is to serve as the workers in an


\textsuperscript{41} Ibid.


\textsuperscript{43} Sandy, Sloane, and Rosentraub, "The Economics of Sport: An International Perspective," 14
organization. The owner decides what course is best for the team to follow. Like individual sports, fans want to see the best players and want to relate to a winning team. Ironically, Szymanski (2003) states, fans also have a tendency to identify themselves with a particular team in their area, even if the team is relatively weak. Poor performance on the field rarely results in a forced movement to a minor league. Therefore, as long as the owner doesn’t allow the team to fold or move, there will always be a profitable team with a substantial fan base.

There is one key difference that allows professional U.S. team sport leagues to be exempt from antitrust laws while still maximizing profit. The leagues stand firm in their dedication towards equalizing the playing strengths of all teams. The success of a league in part is due to the degree of balance among teams. Zimbalist (2003) states that unlike businesses in other industries, league teams not only have to compete against each, but also work in cooperation. The idea of competitive balance within a league brings up the idea of uncertainty.

Uncertainty

Uncertainty is the probability of either team winning a certain event within a league. It typically falls into three categories: match, seasonal, and championship. Match uncertainty is the probability of whether a certain team will win or not. Much work has been done that deals with this uncertainty. Seasonal deals with how close a championship race is within a season. Championship correlates to the possibility of

44 Szymanski, "The Economic Design of Sporting Contests," 1137-87
various champions over a period of years. Very few studies have ever tested either seasonal or championship uncertainty.

Ticket sales are the number one source of revenue for almost all pro sports teams. The price of these tickets can be attributed to many factors including uncertainty, entertainment value, and territory. Match uncertainty has been found to play a large role in the profit maximization of teams. El-Hodiri and Quirk (1971) state that if any one team is “too” superior fans will get bored and not want to view games. With less people attending games, the owners will collect fewer gate receipts. This means owners will only sign contracts with players if the marginal product revenue they add is greater than their cost. If a player makes the team “too” superior he is actually an economic burden on the owner.46 Downward and Dawson (2000) state a team’s marginal revenue will start to fall once it becomes “too” successful.47 However, owners do have to be careful though because attendance is also affected by the probability that the home team will win. Therefore every team has the motivation to be only somewhat superior to other teams.

Entertainment

The overall entertainment provided by a game is also a determinant of attendance and ticket prices. Demmert (1973) says the potential reasons a fan will watch a contest, other than a competitive match, are numerous including rivalries, personalities, and

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personal performances. Social, cultural, and ethnicity all pull certain groups towards specific matches. Some matches are more a matter of pride than competitiveness. From the inner city to national level professional sports rivalries can produce extremely large amounts of attendance. Personality attractiveness is usually the result of a certain player. For example, wide receivers Chad Johnson and Terrell Owens both draw many fans because of their unusual antics and dramatic performances during the game. Lastly, the chances of an unbelievable human performance such as breaking a world record can increase attendance. When Mark McGwire and Sammy Sosa were in a race to break Roger Maris's single season home run record, almost every game was a sell out. Personal reasons to come to a sporting event are always numerous and different, but the more people a team has to draw from the better. This leads to the idea of team territory.

Territory

Territory, defined as the size and demographics of the market area, also influences attendance for each team. The league grants these territorial rights to teams. El-Hordiri (1971) says that the league typically gives exclusive rights to organize a team in a 35-75 mile geographical area around the home playing field. Inside a specific territory there can be multiple differences. Some of the more influential aspects of a territory are the total population and wealth. The more people a territory has and the wealthier the area will draw more people to sporting events. This means owners must take their

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49 Ibid., 12

surrounding territory into effect when reflecting on and making business choices. Sandy (2003) shows this by citing multiple examples of franchises picking up and moving to a new location. Al Davis has moved the Raiders football team 3 times in 20 years, Art Mondell moved the Cleveland Browns to Baltimore, and Jim Irsay moved the Baltimore Colts to Indianapolis. One would think that owners would move to a new territory in order to make a stronger team. However, none of these teams has ever become a consistent powerhouse. Therefore Sandy (2003) concluded it is hard to find any other motive than profit maximization for these relocations.

**Revenue Sharing**

Revenue sharing of ticket sales is another factor that affects the total revenue of a sports team. At each game the revenue generated by all ticket sales is divided between the home and away teams. In some cases both teams receive 50% each, where in others the home team receives more than the away. Szymanski (2003) says revenue sharing causes each team in the league to be dependent on the others in some way. Non-cooperative behavior does not yield joint profit maximization, which makes teams work together to earn maximum profits. Revenue sharing is not only seen in ticket sales. Revenues are also divide in national TV broadcast rights, national league licensing, and other sources.

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51 Sandy, Sloane, and Rosentraub, "The Economics of Sport: An International Perspective," 16
52 Ibid.
53 Ibid., 25
54 Szymanski, "The Economic Design of Sporting Contests," 1137-87
55 Ibid.
The revenue sharing of broadcast rights makes up a large part of team revenue. Demmert (1973) explains that these rights are usually sold by the league in packages consisting of a number of years to broadcast companies. In most U.S. team sport leagues the revenues attained from broadcasts rights are divided equally among all the teams in the league. A problem arises if a team’s games are shown too often on TV because TV viewership is often a substitute for attendance. This would have an adverse effect on ticket sales. Therefore, TV broadcasters show a limited number of games of each team with the majority being road games. This way fans can still watch their team but are more motivated to attend home games, contributing to ticket sales revenue. Kern (2000) says it would be valuable for other leagues to study the NFL’s revenue sharing policies. The NFL divides all revenue earned through national televising (currently averaging 2.2 billion a year) and national licensing equally, splits net ticket revenue 60/40, and shares other revenue sources. Through this system the NFL has achieved unprecedented balance. Kern (2000) also states the current NFL system is by no means perfect and things should be modified because there are relatively no profit incentives for teams to win.

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56 Demmert, "The Economics of Professional Team Sports," 10-15
57 Ibid.
58 Ibid.
59 Ibid.
61 Ibid.
62 Ibid.
63 Ibid.
Incentives

A huge difference between team and individual sports, and also a large reason owners are not inclined to have the best team, is the fact that team sports offer no incentives to win. Szymanski (2003) states that the owners of the team stand to gain basically no direct monetary gain from winning a championship.64 The team may receive a trophy and players can earn substantial bonuses, but the owners don’t get a bonus. It is known that participating in playoffs and winning a championship do increase ticket revenue, merchandise, and sponsorship income. However, revenue sharing takes much of this money away from the owner, which may diminish efforts to win.

Empirical Research

The studies in the area of sports economics are innumerable. They focus on many aspects of sports such as attendance, revenue, and competitive balance. However, few have focused on the idea of profit maximization by teams. Two studies have examined problems directly related to this study of profit maximization. A study by Ferguson, Stewart, Jones, and Dressay (1991) determines if teams maximized profit through the examination of ticket prices.65 Boyd and Boyd (2001) look at home field advantage and the implications it has on ticket pricing.66 Both studies give valuable insights that need to be taken into account in this study.

64 Szymanski, "The Economic Design of Sporting Contests," 1137-87
The work done by Ferguson et al. (1991) is perhaps the most definitive study that deals with profit maximization in professional team sports. This study asserts that both buyers and sellers are influenced by the “fairness” of prices. Obviously, fans do not like it when ticket prices increase, but sellers sometimes need to do this to profit maximize. The question is, will fans will really deny themselves attendance at a favored team’s game just because of a higher ticket price? Or will owners not take the opportunity to make a higher profit? Evidence suggests that the answers to both questions is no. Ferguson et al. (1991) state that attendance of events does not respond differently from regular goods to adjustments in prices. They also say that the pricing difference between various types and packages of seating support the idea that owners use sophisticated practices to earn profits. There has been an ongoing debate on whether professional sports teams are profit. If teams do not profit maximize then the standard economic models should not apply to professional team sports.

Therefore Ferguson et al. (1991) decided to investigate the issue of pricing motivation by looking at the ticket prices of teams in the National Hockey League (NHL). They base their test on the fact that if teams are profit maximizers their pricing behavior will rely on the firm’s demand and cost structure. The first and second order derivative conditions set forth earlier in this paper must also hold true. To evaluate the first fact they used cross-equation restrictions, while the second was tested through numerically examining the derivatives. Ferguson et al.(1991) assumed each team considers fan’s willingness to pay for attendance, then profit maximizes by setting seat

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68 Ibid.
69 Ibid.
prices accordingly to maximize its gate receipts.\textsuperscript{70} Each team will have a different method for price setting based on different territories and team attributes. Therefore Ferguson et al. (1991) believed ticket price was a function of average attendance and a vector of other attributes \((z)\).\textsuperscript{71} All variables for the study are outlined in TABLE 2.3.

**TABLE 2.3**
Variable Definitions for Ferguson et al. (1991) Study

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )</td>
<td>Ticket price</td>
</tr>
<tr>
<td>( A )</td>
<td>Yearly average attendance per game</td>
</tr>
<tr>
<td>( z_1 )</td>
<td>Population of team's home city</td>
</tr>
<tr>
<td>( z_2 )</td>
<td>Per capita income of the team’s home city</td>
</tr>
<tr>
<td>( z_3 )</td>
<td>Dummy=1 if home city is in Canada</td>
</tr>
<tr>
<td>( z_4 )</td>
<td>Number of superstars on the team</td>
</tr>
<tr>
<td>( z_5 )</td>
<td>Teams average ran in league over current season</td>
</tr>
<tr>
<td>( z_6 )</td>
<td>Teams rank in the League at the end of the previous season.</td>
</tr>
</tbody>
</table>

Using these variables, Ferguson et al. (1991) simple model did produce econometric results that offered considerable support for profit maximizing behavior.\textsuperscript{72} All non-sellout teams have a negative second order derivative that was statistically significant. Most sellout teams also have a negative second order derivative but some were insignificant. First order derivatives also had mixed results due to significance levels. However, the study comments on the reliability of the Wald test used for determining significance on three levels: Berndt-Savin inequality, finite sample

\textsuperscript{70} Ibid.
\textsuperscript{71} Ibid.
\textsuperscript{72} Ibid.
distribution, and invariance to reparameterization. A better significance test might lead to results that greatly support profit maximization. Overall, results still show evidence that teams in the NHL do follow a profit maximizing procedure to set ticket prices.

Boyd and Boyd (2001) state that home field advantage plays a major role in ticket prices and profit maximization. Home field advantage is an extremely important aspect in professional sports. In 1990, NFL home teams won nearly 60% of the games and home teams averaged 3.75 more points per game. Also in 1992, 23 out of 26 MLB teams won more games at home than on the road. Since revenue is due in part to winning percentage home field advantage should be taken into account. Therefore Boyd and Boyd (2001) develop a profit maximization model that does home field advantage into account. Since attendance consists of predominantly home team supporters, they believe attendance is a function of ticket prices and the probability the home team will win. They also bring this theory full circle by saying the probability the home team will win is a function of attendance because of the noise and encouragement fans bring. To test their theory Boyd and Boyd (2001) run a regression to evaluate MLB attendance against the six independent variables in FIGURE 2.4 below.

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73 Ibid.

74 Boyd and Boyd, "The Home Field Advantage: Implications for the Pricing of Tickets to Professional Team Sporting Events," 254-64.

75 Ibid.

76 Ibid.
FIGURE 2.4

Equation Stating Variables used in Boyd and Boyd’s (2001) Study

\[ \text{ATTEND} = f(\text{PRICE}, \text{WIN}, \text{WIN}_{-1}, \text{POP}, \text{RECREATE}, \text{INCOME}) \]

TABLE 2.4 illustrates the variable names with their definitions.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>ATTEND</td>
<td>Season home attendance</td>
</tr>
<tr>
<td>PRICE</td>
<td>Average ticket price for season</td>
</tr>
<tr>
<td>WIN</td>
<td>End-of-year winning percentage</td>
</tr>
<tr>
<td>WIN_{-1}</td>
<td>End-of-year winning percentage for previous year</td>
</tr>
<tr>
<td>POP</td>
<td>Population of franchise’s home city</td>
</tr>
<tr>
<td>RECREATE</td>
<td>Alternative forms of local recreation available</td>
</tr>
<tr>
<td>INCOME</td>
<td>Per capita income of franchise’s home city</td>
</tr>
</tbody>
</table>

Results from Boyd and Boyd’s (2001) study show that even though some evidence found raises questions about profit maximizing ticket pricing policies, it in no way disproves teams’ following the profit maximization hypothesis.\textsuperscript{77} Ticket prices are statistically significant in explaining seasonal attendance for teams. Boyd and Boyd (2001) also find simultaneity between winning percentage and attendance.\textsuperscript{78} These results from the model are consistent with team profit maximization. Even though profit maximization in U.S. professional sports teams is far from fully vindicated, these two studies can both be used as evidence and models for future investigations. The more

\textsuperscript{77} Ibid.

\textsuperscript{78} Ibid.
profit maximization theory is supported, the more microeconomic theory can be applied towards professional team sport leagues.

The literature reviewed in this chapter shows that profit maximization is an economic topic that needs to be researched, especially in the world of professional team sports. This chapter not only lays out the fundamentals of profit maximization but also provides a basis for the theory of this paper. It shows that competitiveness, local demographics, and overall entertainment need to be among the data analyzed in further research. Ferguson et al. (1991) and Boyd and Boyd (2001) lay down the groundwork for the use of these variables in empirical research and push for further studies. The following chapter presents an adaptation of Ferguson’s et al. (1991) model as it applies to profit maximization in the NFL.

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79 Ferguson, "The Pricing of Sports Events: Do Teams Maximize Profit?" 297-310

80 Boyd and Boyd, "The Home Field Advantage: Implications for the Pricing of Tickets to Professional Team Sporting Events," 254-64

81 Ferguson, "The Pricing of Sports Events: Do Teams Maximize Profit?" 297-310
CHAPTER III
THEORY

The purpose of this chapter is to lay out the framework for the profit maximization problem in the NFL and interpret the various factors pertaining to it. The beginning of this chapter focuses on the use of profit maximization by NFL team owners. Next, a mathematical model is outlined that may be used to test the hypothesis of profit maximization in the NFL. The second half of this chapter goes into great depth in discussing the theoretical variables affecting profit maximization in the NFL.

This paper extends the research and model created by Ferguson, Stewart, Jones, and Dressay (1991) when analyzing ticket prices and profit maximization in the NHL.¹ Their study is the main source for the model constructed in this research. The models and variables presented in this chapter will be empirically tested in Chapter IV.

Profit Maximization in the NFL

Are NFL teams profit maximizing companies, where owners care solely about making money? Scully (1995) states that all teams make profit in the NFL, but that does not answer the question. Most empirical research on profit maximization has been done in industries other than sports, where profit maximization is known to be the main goal. A study by Tauer (1995) and another by Featherstone & Rahman (1996) focused on the agricultural industry while Levitt (2006) examined part of the food industry. Testing the profit maximization behaviors of NFL team owners will not only contribute to existing literature on profit maximization but also illuminate the business of U.S. team sports.

In professional U.S. team sports many owners claim to have different motives than simply business. Most people assume professional sports teams are businesses and owners make decisions about their team in order to maximize profits. However, others suggest teams are not currently profit maximizing. They point to scalpers or ticket agents selling tickets at prices in excess of face value and constant sellouts as evidence that teams could raise prices to increase profits. For this reason, a profit maximization model is necessary to test if owners profit maximize or use different motives to determine what

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is best for the team. The profit maximization models used to test other industries can be adapted to the NFL to determine how owners behave while making business decisions.

If owners profit maximize, they must pay close attention to the revenues generated and costs incurred throughout the season. The revenue a team receives comes in two main forms. The majority of revenue is collected from the gate receipts of each game. Secondary forms of revenue arise from the revenue sharing of NFL broadcasts and merchandise sales. Even though revenue sharing from NFL broadcasts does make up a fairly significant part of a team’s total revenue, it is fixed amount. Szymanski (2003) states that the NFL signs contracts with national and local broadcasting agencies. The revenues generated by these contracts are divided equally among the teams. The teams have virtually no control in the revenue they receive from league sharing. Therefore, owners focus on gate receipts, which they can control. Owners can set ticket prices at any level to maximize profits for the season. By adjusting ticket prices to fan willingness to attend a game, owners can maximize gate receipts.

The NFL differs dramatically from other U.S. professional sports leagues. Because the research of Ferguson et al. (1991) is the basis of this study, it is essential to elaborate on these differences, especially between the NHL and the NFL. These differences could affect owner’s decisions on ticket pricing and profit maximizing behavior. The main difference between the two leagues is the number of games each team plays in a season. An NFL team has only one game each week, with a total of 16 games in a year. The NHL has 82 games a year with multiple games each week. The

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7 Ferguson, "The Pricing of Sports Events: Do Teams Maximize Profit?" 297-310
limited number of games the NFL offers should increase the fans’ demand for attendance. With the NHL’s 82 games a year, there are plenty of alternatives for fans. If they cannot attend a certain game, the chance of them attending a different game is dramatically higher. If the overall attendance were equal for the entire year, the NHL would still average fewer fans at each individual game because attendance would spread itself out. Another difference is the anticipation for each season. The NFL is the most popular professional sport in the U.S. and its season is highly anticipated. The NHL’s season is not highly anticipated, with the exception of Canadian teams. High anticipation of the NFL season should also increase attendance demand throughout the entire season.

The Profit Maximization Problem

Profit maximization, as discussed earlier, implies that owners of firms make business decisions in order to produce the maximum profit possible. NFL teams are definitely profitable organizations, but do they strive to make the most money possible? Ferguson et al. (1991) study of profit maximization in the NHL will be adapted and manipulated to test the profit maximizing behaviors of team owners in the NFL.\(^8\)

Following the model Ferguson et al (1991) use, it is assumed that the costs varying with attendance at each individual game are so small that operating costs and stadium costs should not play a large role in profit maximization.\(^9\) The majority of costs for each team consist of expenses made towards players such as salary, per diem, etc. Also, since all revenue collected from NFL labeled merchandise and the NFL nation

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\(^8\) Ibid.

\(^9\) Ibid.
television package are divided equally among the teams, they are not factored into this model either. This implies owners' sole objective is to maximize the difference between total gate receipts in each year and total player expenses. To do this owners will adjust ticket prices until they find a price that returns maximum profit, \( \pi \), given the amount spent on players. The revenue generated by gate receipts is simply ticket price, \( p \), multiplied by attendance, \( A \). Therefore, the profit maximization formula for NFL owners is expressed in equation 3.1,

\[
\text{Max } \pi = pA - E \quad \text{subject to } A \leq C
\]

where \( E \) represents total player expenses. Unlike many other industries, there is a constraint on the level of output available for NFL owners. The maximum number of people that can attend any game is limited to the capacity, \( C \), of the stadium.

Next, ticket price must be looked at as more than just a variable. Ticket price itself is a function relying on many other variables. Like the research of Ferguson et al (1991), this study will define ticket price as a function of attendance and a vector of other variables.\(^{10}\) However, it will also incorporate player expense as a determinant of ticket price. Attendance should have a direct relationship with price. If attendance is too low owners need to lower ticket prices in order to stimulate more people to attend games. On the other hand, if games are consistently selling out, with excess people still willing to buy tickets, prices should be raised until there is no excess demand for tickets. The ticket price function is expressed in equation 3.2,

\[
p = f(A, E, z; \theta)
\]
where $z$ is the vector of exogenous attributes of the team and $\theta$ is the vector parameters. The attributes belonging in $z$ can have both positive and negative correlation with price, and will be discussed later in this chapter. Equation 3.2 presents some problems. It is believed that both attendance and player expenses are endogenous variables. This means instrumental variables are needed to make the regression unbiased and consistent. A Two-Stage Least Squares model is used to incorporate instrumental variables. The instrumental variables will produce theoretical values for attendance and player expenses, which will be called $\hat{A}$ and $\hat{E}$ respectively. Therefore, the inverse demand function from equation 3.2 can be represented by equation 3.3,

$$p = \beta_0 + \beta_1 \hat{A} + \beta_2 \hat{E} + \beta_k z$$

where all $\beta$’s represent the constant or coefficients of the different determinants of price in the 2nd stage of the Two-Stage Least Squares model. By substituting the right side of equation 3.3 for $p$ in equation 3.1, equation 3.4 is obtained.

$$\pi = \hat{A}^* (\beta_0 + \beta_1 \hat{A} + \beta_2 \hat{E} + \beta_k z) - \hat{E}$$

Simplifying creates the profit function presented in equation 3.5. Since profit maximization corresponds to revenue minus costs, equation 3.5 will be the basis for all profit maximizing testing.

$$\pi = \hat{A} \beta_0 + \hat{A}^2 \beta_1 + \hat{A} \beta_2 \hat{E} - \hat{E} + \hat{A} \beta_k z$$

Profit maximization can only occur where the first order derivative equals zero. Therefore, the derivative of profit must be taken with respect to attendance. Equation 3.6 shows this first order derivative.

$$\frac{\partial \pi}{\partial \hat{A}} = \beta_0 + 2 \beta_1 \hat{A} + \beta_2 \hat{E} + \beta_k z = 0$$
By separating the $2\beta_1\hat{A}$ term and rearranging equation 3.6 we achieve equation 3.7.

$$\frac{\partial \pi}{\partial A} = \beta_0 + \beta_1\hat{A} + \beta_2\hat{E} + \beta_kz + \beta_1\hat{A} = 0$$  \hspace{1cm} 3.7

Referring back to equation 3.3 we are able to substitute $p$ into equation 3.7 to form equation 3.8.

$$\frac{\partial \pi}{\partial A} = p + \beta_1\hat{A} = 0$$  \hspace{1cm} 3.8

Finally, the second order derivative must be solved for in order to test the second condition of profit maximization. The condition states that the second order derivative must be negative to be profit maximizing. The derivative of $\partial\pi/\partial\hat{A}$ must be taken with respect to attendance. Equation 3.9 shows the second order derivative.

$$\frac{\partial^2 \pi}{\partial A^2} = 2\beta_1 \leq 0$$  \hspace{1cm} 3.9

As noted previously, stadium capacity places a restriction on the number of people attending a game. This constraint must be taken into account. To do this a univariate Kuhn-Tucker approach is used. A Kuhn-Tucker approach is used because there is a linear function with a ceiling restriction placed on attendance (APPENDIX A). A maximum profit must be generated between zero and stadium capacity. Therefore at the profit maximizing attendance, the following, presented in equation 3.10, must hold true.

$$\frac{\partial \pi}{\partial A} \geq 0; C - \hat{A} \geq 0; \frac{\partial \pi}{\partial A} \times (C - \hat{A}) = 0$$  \hspace{1cm} 3.10

These conditions imply the capacity constraint is binding unless local concavity exists. To incorporate these constraints, this research performs estimation on a system of equations. However, since all of equation 3.10 can't be used, this study uses the last
condition to represent the restriction on owners' choices. This restriction is presented in equation 3.11.

\[ \frac{\partial \pi}{\partial \hat{A}} \times (C - \hat{A}) = (p + \beta_1 \hat{A}) \times (C - \hat{A}) = 0 \tag{3.11} \]

Ferguson et al. (1991) use the same restriction approach in their research.\textsuperscript{11}

By performing estimation on the following system of equations derived from equations 3.2 and 3.11,

i) \[ p = \beta_0 + \beta_1 \hat{A}_i + \beta_2 \hat{E}_i + \beta_k z_i + \varepsilon_i \]

ii) \[ (p + \beta_1 \hat{A}) \times (C - \hat{A}) = \varepsilon_{2i} \]

where \( i \) represents the observation, the effects of the theoretical determinants on ticket prices in the NFL will be determined. Using these coefficients and determinants, the first and second order derivatives can be numerically calculated. The results of these calculations should provide insight on whether NFL team owners behave in profit maximizing ways.

Since the equations to determine profit maximization have been set forth, it is important to consider the exogenous variables that affect ticket prices and in turn profit. Owners must take these variables into account when setting ticket prices in order to be profit maximizing.

\textsuperscript{11} Ibid.
Theoretical Determinants of Profit Maximization in the NFL

This section will discuss the various factors thought to influence the profit maximization behaviors of owners in the NFL. These exogenous variables will make up the attributes found in the z vector mentioned in the previous section. Some variables change from year to year, while others stay relatively constant. Based on the literature presented in Chapter II, it is expected that various territorial, team, and league variables all play a role in profit maximization through ticket pricing of NFL games. Figure 3.1 provides an overview of these factors. Each of these variables will then be discussed in detail thereafter.
 Territory

Territory is the market area’s size and characteristics that a team has to draw potential fans and revenue from. Like Ferguson et al (1991), this research needs to take
aspects of a team’s territory into account. Particular demographics of a territory may possibly have a correlation with the revenue of an NFL team. The total population of a territory should directly affect the number of fans wanting to attend the home games. This creates a greater demand for tickets, allowing owners to raise prices, leading to greater revenue. Ferguson et al (1991) find a positive correlation between the population of the home team’s city and ticket prices in the NHL. Therefore, based on the findings of Ferguson et al. (1991), this research hypothesizes that population and ticket price will have a positive correlation in the NFL.

Also important to consider in a team’s territory is wealth. The wealthier people are, the more willing they are to buy tickets to NFL games. Coates found that income is statistically significant in determining the number of season tickets purchased for sporting events but insignificant in single game tickets purchased. This means income does affect the demand for the number of tickets purchased, but perhaps only for season tickets. Ferguson et al. (1991) find per capita income and ticket prices to be positively correlated in the NHL, showing attending NHL games to be a normal good. However, Boyd and Boyd (2001) find a negative correlation between income and attendance in the MLB, meaning attending MLB games is an inferior good. Is NFL attendance a normal

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12 Ibid.
13 Ibid.
14 Ibid.
16 Ferguson, "The Pricing of Sports Events: Do Teams Maximize Profit?" 297-310
or inferior good? This research sides with the findings of Ferguson et al (1991) in hypothesizing that measures of wealth will have a positive correlation with ticket prices because NFL tickets are a normal good.

One last characteristic to take into account is the alternative recreation and entertainment a territory has to offer. NFL football games are a source of entertainment and compete with other local entertainment for people’s time and money. Many times, these alternative forms of recreation are defined as the number of other professional sports teams the territory is host to. There are opposing arguments to the effects multiple sports teams in a single territory create. One side says multiple sports teams act as substitutes for each other. People will spread their attendance out between all the teams. If a certain city has more than one professional sports team, people may attend fewer NFL home games. As a result, demand for tickets should fall, leading to lower revenue. Boyd and Boyd’s (2001) research finds a negative correlation between recreation and MLB attendance, supporting the substitute theory.18 However, the opposite approach implies that multiple sports teams can act as complements. Cities may pride themselves on their sports teams. They see their sports teams as an extension of their territory and support every team. Due to these conflicting theories, it is uncertain what type of impact alternative forms of recreation will have a on the ticket prices and revenue in the NFL.

Uncertainty

The Uncertainty of Outcome Hypothesis states that fan demand for attending sporting events is positively related to the outcome uncertainty, in that fans prefer to see

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18 Ibid.
the home team be victorious but only after a competitive match.\textsuperscript{19} It is not clear how uncertainty in the NFL affects ticket prices. However, like Szymanski (2003) says, fans have a tendency to strongly identify themselves with a particular team even if the team is weak.\textsuperscript{20} Therefore uncertainty must be broken down into two categories: team success and competitive balance.

Team success in the NFL should affect the ticket prices owners set. Even though fans are willing to support a weak team, they would rather support a winner. This is good news for owners. Even if the product they put on the field is weak they can still make a profit. The studies of both Ferguson et al (1991) and Boyd and Boyd (2001) acknowledge that team success, both past and present, affects ticket prices and attendance. The research of Ferguson et al (1991) shows a positive correlation between winning percentage and ticket prices.\textsuperscript{21} Boyd and Boyd (2001) also show a positive correlation between winning percentage and attendance.\textsuperscript{22} Both studies incorporate the current and previous year’s win percentage into their models. People prefer to watch winning teams because they attract attention. In any given year, the higher the win percentage, the more people will want to attend games. The performance of a team in the previous year can affect attendance for the current year. If a team did well last year, people will expect another good season and will want to attend more games. Also, people who were not previously fans might now be attracted to the team. Both these factors increase the

\begin{flushleft}
\textsuperscript{19} Rodney Fort and Young Hoon Lee, "Fan Demand and the Uncertainty of Outcome Hypothesis," Working paper, Washington State University.

\textsuperscript{20} Szymanski, "The Economic Design of Sporting Contests," 1137-87

\textsuperscript{21} Ferguson, "The Pricing of Sports Events: Do Teams Maximize Profit?" 297-310

\textsuperscript{22} Boyd and Boyd, "The Home Field Advantage: Implications for the Pricing of Tickets to Professional Team Sporting Events," 254-64.
\end{flushleft}
demand for attendance and can lead to changes in ticket prices. In both cases, past and present, this research hypothesizes that win percentage of a team will have a positive correlation with ticket prices.

Win percentage has its limitations, though. Like Downward and Dawson (2000) state, the profits a team achieves can start to fall if the team becomes "too" successful.23 If a team becomes so good that opposing teams have little chance of winning, fans will become bored and not want to attend games. The same applies for teams who are so bad they consistently lose. Therefore it is necessary to take the competitive balance of the league into account. Competitive balance is a measure of how evenly wins are distributed to teams throughout the league. El-Hodiri and Quirk (1971) state that predictable outcomes in sporting events lower the attendance of the event; the greater the certainty of an outcome, the lower the attendance.24 The greatest fan interest occurs when the home team has only a slight advantage over the opponent.

In the NFL, divisional games are oftentimes of more importance to the teams and fans. Historical rivalries between divisional opponents create a strong sense pride for teams and fans, increasing the demand for attendance. Also, divisional games often entail playoff implications that increase fan support and attendance. Therefore the unique effects inside each division must be taken into account within a profit maximization model. In this study it is not clear how each division's effects will correlate with ticket prices.


Past empirical research on profit maximization has not taken competitive balance into account. This study will attempt to include the competitive balance of the NFL in its profit maximization model. The model will use the Herfindahl-Hirschman Index (HHI) to approximate the impact of competitive balance on the NFL. The HHI is a measure of how equally wins are distributed among teams in the league. The Herfindahl-Hirschman Index is most commonly used to examine market concentration in industrial organization, but it is also applicable to competitive balance.\(^{25}\) The dHHI, or deviation from the "most equal distribution of wins", will be calculated but not used because the HHI has a wider range of values. Based on literature presented in Chapter II, it is logical to hypothesize that higher levels of competitive balance, meaning a lower HHI, will allow for higher ticket prices and revenue through increased demand for attendance.

**Star Power**

Every team in the NFL has individual players who draw fans to games just to get a glimpse of their skills and talent. These players rise above the rest in ability to entertain and awe the crowd with their remarkable feats of athleticism. Berri, Schmidt, and Brook (2004) suggested an organization could shift the focus from a team's poor performance towards individual star players in order to increase attendance.\(^{26}\) They players are typically considered stars because they are the most talented players on the field. Obviously, there are a few players who draw fans based on other credentials than


talent. Fans place great expectations on highly touted rookies and attend games to see if these newcomers live up to the hype. These players are not proven stars yet, but they do create entertainment for the fans. Also, players who have unique attitudes and strange antics draw fans to the game. People will attend games to see a dramatic sideline conflict or on-field antics. These events make the game more enjoyable for fans. This study assumes star power of a team can be measured by the number of players representing the team at that year's Pro-Bowl. These players' "star" abilities are what draw some fans to a game. Berri, Schmidt, and Brook (2004) found star power to be significant in raising attendance demand, however winning percentage had a much greater effect. Therefore it is expected that the star power of a team will have a positive correlation with ticket prices. NFL teams with high star power will have higher demand for attendance, allowing owners to increase ticket prices.

Conclusion

This chapter has developed the theory and mathematics behind profit maximization in the NFL to be tested in the upcoming chapters. The goal of testing is to determine if owners of NFL franchises make decisions regarding ticket prices that are consistent with profit maximizing behavior. All testable implications will be outlined and examined in the following chapter through the use of a detailed data set. The data set and a description of its variables and sources will also be provided in the next chapter.

The determinants of ticket prices in the NFL will be tested for each team over a three-year period. Population, income, team success, star power, and player expenses are

27 Ibid.
hypothesized to have a positive relationship with ticket prices. Attendance, alternative recreation, and the HHI measure of competitive balance are expected to have a negative relationship with ticket prices. The empirical model constructed in the next chapter will examine these hypotheses. However, numerical analysis beyond the empirical model will determine if owners set ticket prices in a profit maximizing fashion. By applying the results of the empirical model, the profit maximizing conditions shown earlier will be tested. This testing is the focus of this paper and will explain whether NFL franchises act as profit maximizing businesses.
CHAPTER IV
DATA AND METHODS

This chapter describes the data set that has been compiled in order to test the model developed in the previous chapter. This chapter discusses each individual variable in the model. The dependent variable will be discussed first, followed by the other endogenous variables, and finally the exogenous variables. Lastly, the econometric methodology for testing the empirical model will be discussed.

The data takes into account all 32 NFL football teams over three regular seasons from 2003 to 2005. The unit of observation is a single regular season for each team. Over the three seasons there are a total of 96 observations. The dependent variable is average ticket price and there are 19 independent variables. All the statistics in question are recorded in this data set. FIGURE 4.1 expresses the empirical model for this research and TABLE 4.1 gives a complete list of all the variables.
FIGURE 4.1
Simultaneous Empirical Model for Ticket Prices in the NFL

\[ \text{PRICE} = f(\text{ATTEND}, \text{POP}, \text{WEALTH}, \text{PWIN}, \text{CWIN}, \text{ALTREC}, \text{STAR}, \text{PEXP}, \text{HHI}, \text{DIVISIONAL DUMMY VARIABLES, YEAR DUMMY VARIABLES}) \]

\[ \text{s.t. } [\text{PRICE} + \beta_1 \text{ATTEND}] [\text{CAP-ATTEND}] = 0 \]

TABLE 4.1
Variable Definitions

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRICE</td>
<td>Average game ticket price for season in 2003 dollars</td>
</tr>
<tr>
<td>ATTEND</td>
<td>Season home attendance</td>
</tr>
<tr>
<td>POP</td>
<td>Population of franchise’s home city</td>
</tr>
<tr>
<td>WEALTH</td>
<td>Median income of franchise’s home city in 2003 dollars</td>
</tr>
<tr>
<td>PWIN</td>
<td>End-of-year winning percentage for previous year</td>
</tr>
<tr>
<td>CWIN</td>
<td>End-of-year winning percentage</td>
</tr>
<tr>
<td>ALTREC</td>
<td>Alternative forms of local recreation available in home city</td>
</tr>
<tr>
<td>STAR</td>
<td>Number of Pro Bowl selections</td>
</tr>
<tr>
<td>PEXP</td>
<td>Total player expenses in 2003 dollars</td>
</tr>
<tr>
<td>HHI</td>
<td>Yearly HHI in the NFL</td>
</tr>
<tr>
<td>CAP</td>
<td>Stadium Capacity</td>
</tr>
<tr>
<td>NFCE</td>
<td>Dummy for division NFC East</td>
</tr>
<tr>
<td>NFCN</td>
<td>Dummy for division NFC North</td>
</tr>
<tr>
<td>NFCS</td>
<td>Dummy for division NFC South</td>
</tr>
<tr>
<td>NFCW</td>
<td>Dummy for division NFC West</td>
</tr>
<tr>
<td>AFCE</td>
<td>Dummy for division AFC East</td>
</tr>
<tr>
<td>AFCN</td>
<td>Dummy for division AFC North</td>
</tr>
<tr>
<td>AFCS</td>
<td>Dummy for division AFC South</td>
</tr>
<tr>
<td>YEAR03</td>
<td>Dummy for 2003 season</td>
</tr>
<tr>
<td>YEAR04</td>
<td>Dummy for 2004 season</td>
</tr>
</tbody>
</table>
**Dependent Variable**

The dependent variable in this model is the single game average ticket price to attend an NFL football game (PRICE). Since ticket prices for club and premium seats could not be obtained, an average ticket price was used to represent all possible seats at an NFL game. Average ticket prices were compiled for every team in the NFL during the 2003 season through the 2005 season. These values were found at Team Marketing Report’s Fan Cost Index, which defined average ticket price as a weighted average of season ticket prices for general and club-level seats. Values were determined by factoring the tickets in each price range as a percentage of the total number of seats in each stadium. A simple example would be if 40,000 seats in a 60,000-seat stadium were upper level seating with a ticket price of $20 and the remaining 20,000 seats were lower level seating with a ticket price of $40. Average ticket price would be calculated as follows:

\[ \text{PRICE} = \frac{40000}{60000} \times 20 + \frac{20000}{60000} \times 40 = 26.67 \]  

Luxury suite sales were excluded from the number. Prices were then adjusted to 2003 dollars using the Consumer Price Index.

This study examines the effects that potential determinants have on the pricing of average ticket prices of NFL football games then applies them to the first and second order derivatives in order to test for profit maximization. The goal is to determine if NFL owners make profit maximizing decisions on ticket pricing by taking into account team statistics, league statistics, and territorial characteristics.
Other Endogenous Variables:

Attendance

The attendance for NFL football games is examined as a determinant of ticket pricing in the NFL. Since attendance is believed to be an endogenous variable in the regression it is therefore dependent on the exogenous variables. The determinants that affect average ticket price also affect attendance, causing a significant correlation between the two variables. Attendance is defined as the total number of spectators at an NFL team's home games for a given year. Attendance was found at NFL.com for each of the 32 teams in the NFL. The expectation is that attendance will have a negative relationship with price.

Player Expenses

The costs of running a professional football team are not small. Most of these costs are either operating expenses or player expenses. This study assumes operating costs, such as stadium operation and general management costs, do not vary by large amounts across NFL teams. Since these costs are assumed to be fixed, and that data for them is not readily available, they are excluded from the model. However, expenses such as player and coach salaries, benefits, and bonuses do differ throughout the league. Even though there is a salary cap in the NFL, player expenses (EXP) vary significantly for each team. Therefore, it is necessary to take this into account in the model. Like attendance, player expenses are believed to be endogenous in the model. Player expense data was collected from the NFL Team Valuations conducted by Forbes. They were then adjusted
to 2003 dollars using the Consumer Price Index. It is expected that average ticket price and player expenses will be positively related.

**Exogenous Variables:**

**Demographic Effects**

The demographics in each city represented by a NFL team are potential determinants of NFL ticket prices and other endogenous variables. Thus population and wealth of a city are relevant to this study. These variables represent the market size and characteristics each NFL team has to operate in. Both the population (POP) and median income (WEALTH) for each of the 32 NFL territories were collected from the US Census Bureau’s 2000 census and defined as a variable in the data set. Median income was adjusted to 2003 dollars using the Consumer Price Index. It is expected that both population and median income will relate positively with NFL ticket prices.

**Alternative Recreation**

Alternative forms of recreation are also included as a determinant of ticket prices and other endogenous variables. The number of different professional sports teams in each territory (ALTREC) represents the possible substitutes for attending an NFL football game. This data includes teams in the four major professional sports leagues of the U.S.: MLB, NBA, NHL, and NFL. The number of other professional sports teams in an individual territory was found for all 32 NFL teams. As the number of substitutes for a particular good goes up, the price of that good should go down in order to stay
competitive. Therefore, it is expected that alternative recreation will have a negative relationship with price.

**Team Success**

Team statistics are also relevant to the endogenous variables. Data was collected on an individual basis for all 32 teams in the NFL for the 2003 season through the 2005 season.¹

The success of an individual team is incorporated into the model. Research by Ferguson, Stewart, Jones, and Le Dressay (1991) shows that the previous and current rank in the league are a determinant of ticket prices in the NHL.² This study will use winning percentage instead of rank to represent team success. The annual winning percentage is found by dividing the number of games won by the number of games played. Both the current year’s winning percentage (CWIN) and the previous year’s winning percentage (PWIN) were collected for each team over all three years. These variables will show how current and prior success of an individual NFL team affect current ticket prices. A higher winning percentage in both past and current seasons should draw more fans to the games. Both variables are expected to have a positive relationship with ticket prices.

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¹ All season’s previous and current winning percentages were found at ESPN.com

Star Power

The talent and entertainment provided by players on the team is relevant to the model. Each team has individual players that provide some type of entertainment that draws fans to a game. The NFL Pro Bowl is a game played after the season ends. Fans, players, and coaches all vote for their favorite players at each position. Players with the most votes are selected to represent their conference in the NFL Pro Bowl. Typically these players are regarded as the best players in the league. Therefore, it seems logical that the players voted into the Pro Bowl from an individual team (STAR) will draw fans to games. Their performances during the current season provide greater amounts of entertainment for the fans at the games. Ferguson et al. (1991) found that the number of superstars on a team in a given year increases the ticket prices for that team.\(^3\) Therefore, this study expects the number of Pro Bowl players on a local team to positively relate to ticket prices in that same year.

Competitive Balance

The Herfindahl Hirschman Index (HHI) is used as a measure of competitive balance in the NFL. The Herfindahl Hirschman Index measures the total number of wins in a season by an individual team in comparison to the total of number of wins throughout the entire league (HHI). The dHHI, or \(HHI - \frac{1}{N}\) where \(N\) is number of teams in the league, was also calculated but not used because \(N\) was constant across all three seasons allowing the HHI to produce a greater range of values. In a season where some teams win all the time and others lose all the time, the HHI will be close to one. If wins

\(^3\) Ibid.
are divided evenly throughout the league, then the HHI will be zero. FIGURE 4.2 shows how the HHI is derived from the data set. \( W_t \) is the number of games won by team \( t \), \( N \) is the number of teams in the league, and \( G \) is the number of games played by each team during a season.

**FIGURE 4.2**

Equation for Herfindahl Hirschman Index

\[
HHI = \frac{\sum_{i=t}^{N} 2W_t}{NG}
\]

If the NFL does not have competitive balance, resulting in a high HHI, then the demand for attending games should fall. A more equal distribution of wins should increase demand for attendance and raise ticket prices. Therefore it is expected that HHI should have a negative relationship with ticket prices.

**Division**

There are eight divisions in the NFL with four in both the National Football Conference and the American Football Conference. Different divisions have specific effects on the endogenous variables, which are accounted for through dummy variables. Every division except one will have a dummy variable representing it (NFCE, NFCN, NFCS, NFCW, AFCE, AFCN, AFCS). These variables take on a value of 1 if the team being observed is part of that division. Otherwise it will have a value of 0. It is uncertain how these dummy variables will relate to ticket prices.
Year

Every year brings about unforeseen changes due to external forces that could factor into the endogenous variables. An example would be hurricane Katrina causing the New Orleans Saints to play in Baton Rouge and San Antonio for the 2005 season. These factors will be taken into account by using a dummy variable to represent two of the three years being observed (YEAR03, YEAR04). A value of 1 is given to any observation occurring in that year, otherwise it is 0. The effect that the years will have on ticket prices is uncertain.

Capacity

The total number of people allowed to attend a NFL football game has a ceiling restriction placed on it. Every stadium in the NFL has a certain capacity that it theoretically cannot exceed. The total number of seats in each stadium (CAP) will therefore be used as a restriction in the model. Using capacity allows the Kuhn-Tucker conditions to be enforced through the restriction equation. Ferguson et al. (1991) used the same approach in their study.\(^4\) However, data showed that some teams had total attendance greater than stadium capacity. If this was the case, capacity was increased so that total attendance equaled stadium capacity. With these teams now considered sell out teams, the ceiling constraint of the Kuhn-Tucker condition becomes binding. Therefore, depending on the observation, either the internal or ceiling constraint will be binding. Since capacity is only used as a restriction, its effects on ticket prices will not be measured.

\(^4\) Ibid.
Econometric Methodology

The data was then regressed to begin the process of testing profit maximization in the NFL. First, an Ordinary Least Squares (OLS) estimator was used to determine the individual impact and significance of the truly exogenous variables on both the dependent and endogenous variables. Any endogenous variables would be correlated with the residuals if an OLS estimator was used. Therefore, instrumental variables that are both correlated with the endogenous variables and uncorrelated to the residuals were used. These variables eliminated the correlation between endogenous variables and the error terms. Using the results from OLS regressions the instrumental variable for ATTEND was determined to be POP. PEXP was found to be insignificant when regressed against PRICE, and was therefore uncorrelated with the residuals. PEXP was used as an exogenous variable from then on.

Next, in order to incorporate both the instrumental variable and the restriction equation, a System Two-stage Least Squares (STSLs) regression approach was used. First, an initial regression equation was specified where the dependent variable is determined by the endogenous and exogenous variables. STSLs then performed an OLS estimation on the endogenous variables versus an instrument list. The instrument list was comprised of all the exogenous variables from the initial equation and also the instrumental variable decided upon earlier. This regression found the portions of the endogenous and exogenous variables that could be attributed to the instrumental variable. The fitted values produced by this regression were saved for later use.

In the second stage of STSLs regression, another OLS regression is performed. In this OLS regression the initial equation was estimated. However, the endogenous
variable was replaced by the fitted values obtained in the previous step. Also, the restriction equation is simultaneously applied to enforce the cross equation restrictions. Therefore, the results of this regression were the coefficients that best estimated both the initial equation and the restriction equation.

Numerous adjustments were made to improve the fit of model and significance of the independent variables. The normality of the residuals was an econometric issue for the regression. The Jarque-Bera statistic indicates normality problems. If the Jarque-Bera stat is greater than the critical chi-squared value of 5.99 for the 5% significance level, then the residuals have a non-normal distribution. If the residuals don’t have a normal distribution, the t-statistics for the model will not be dependable. Therefore, in order to insure normality, the Arizona Cardinals were found to be an outlier and were removed from the model. This was due to the fact that their average attendance was only half that of the next closest team. While most teams came reasonably close to selling out each season, the Arizona Cardinals barely filled half the stadium. Removing this outlier corrected the non-normality of the error term, producing a Jarque-Bera statistic below 5.99.

Another measure necessary to improve estimation was the removal of the HHI measure of competitive balance. HHI was dropped from the model because it created a near singular matrix when used as a determinant. A near singular matrix occurs when the determinant of a matrix used in the regression process is equal to zero. This happens when one row (or column) of the data set is a multiple of some other rows (or columns). Since the determinant of the matrix is zero, the inverse of that matrix is undefined.
Therefore, because of the undefined inverse matrix, EViews was unable to estimate the regression.

The final models decided upon used \( \text{PRICE}^x \) as the dependent variable in order to achieve the best fit while maintaining the significance of many variables. This exponent changed the entire derivation of the profit maximizing conditions and the restriction equation presented in chapter 3. To correct for this, a new set of profit maximization conditions and restriction equations were derived and incorporated into the model's specification. (See APPENDIX B)

Lastly, a numerical analysis was performed in order to test the two profit maximizing conditions. These conditions were tested by substituting the appropriate observation values and STSLS partial coefficients into the derived profit maximizing conditions. These results were then checked for significance by conducting a t-statistic test. Since numerical evaluation did not automatically produce t-statistics like STSLS, the t-statistics for the first and second-order derivatives had to be derived by hand and numerically calculated. When developing these t-statistics the methods outlined in Goldberger (1964) were used to calculate the standard error for each derivative.\(^5\) The Delta Method, outlined in Davidson and McKinnon, was also used in order to approximate the standard error of a nonlinear function that occurred in the second-order derivative’s t-statistic.\(^6\) The Delta Method incorporates Taylor’s Theorem in order to produce a linear approximation of the nonlinear function, from which the standard error


of the original nonlinear function can be approximated. The derivation of these t-statistics and the Delta Method is outlined in APPENDIX C.

This chapter has provided an in depth description of all the relevant variables in this research project. It also explained and expanded on the methodology used to analyze the model. The following chapter will describe the results provided from the regression and numerical analysis. It will also draw conclusions from the results concerning the theory of profit maximization.
CHAPTER V
RESULTS AND CONCLUSIONS

This chapter will examine the results produced by both the regression and numerical analysis of the data described in the previous chapter. The first section of this chapter will focus on the results of the system Two-Stage Least Squared regression (STSL). The second section will examine the numerical analysis of the profit maximization conditions through application of the STSL regression. The final section will outline and develop any conclusions that may be drawn from these empirical results. It will also examine any shortcomings of the research and put forth ideas for further studies.
Results:

System Two-Stage Least Squares

The following table (TABLE 5.1) summarizes the STSLS regression results for the determinants of ticket prices in the NFL. The t-statistics are displayed in parentheses below each coefficient. A blank cell represents the omission of a variable from the model. Models one through three are the STSLS regression results with various combinations of independent variables and exponential factors of the dependent variable. All three models were used when determining the best estimation for the average ticket price of an NFL team (PRICE). In each model the partial coefficient of attendance is the only restricted coefficient. The restriction equation satisfies the Kuhn-Tucker conditions and is altered in each model due to the changes that occur in first order derivatives. The code for running each regression in Eviews is presented in APPENDIX D.
TABLE 5.1
System Two-Stage Least Squared Regression Results
The Determinants of Ticket Price

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>Exponent value that PRICE was raised by</td>
<td>1</td>
<td>1.76</td>
<td>1.8</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td>66.82</td>
<td>890.99</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(4.74)*</td>
<td>(1.48)</td>
<td>-</td>
</tr>
<tr>
<td>ATTEND</td>
<td>Season home attendance (endogenous)</td>
<td>-9.99E-05</td>
<td>-0.0023</td>
<td>-0.0009</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-55.22)*</td>
<td>(-2.07)*</td>
<td>(-2.06)*</td>
</tr>
<tr>
<td>POP</td>
<td>Population of franchise's home city</td>
<td>IV</td>
<td>IV</td>
<td>IV</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>WEALTH</td>
<td>Median income of franchise's home city in 2003 dollars</td>
<td>5.81E-05</td>
<td>0.0021</td>
<td>0.0026</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3.38)*</td>
<td>(5.68)*</td>
<td>(6.75)*</td>
</tr>
<tr>
<td>PWIN</td>
<td>End-of-year winning percentage for previous year</td>
<td>9.49</td>
<td>364.43</td>
<td>446.04</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.64)</td>
<td>(3.05)*</td>
<td>(3.39)*</td>
</tr>
<tr>
<td>CWIN</td>
<td>End-of-year winning percentage</td>
<td>4.86</td>
<td>200.06</td>
<td>229.74</td>
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<td></td>
<td></td>
<td>(0.69)</td>
<td>-1.36</td>
<td>(1.90)</td>
</tr>
<tr>
<td>ALTRECV</td>
<td>Alternative forms of local recreation available in home city</td>
<td>2.31</td>
<td>77.21</td>
<td>93.67</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.50)*</td>
<td>(3.95)*</td>
<td>(4.55)*</td>
</tr>
<tr>
<td>STAR</td>
<td>Number of Pro Bowl selections</td>
<td>0.08</td>
<td>-0.11</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.11)</td>
<td>(-0.01)</td>
<td>-</td>
</tr>
<tr>
<td>PEXP</td>
<td>Total player expenses in 2003 dollars</td>
<td>3.54E-09</td>
<td>4.57E-07</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.031)</td>
<td>(0.19)</td>
<td>-</td>
</tr>
<tr>
<td>HHI</td>
<td>Yearly HHI in the NFL</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>CAP</td>
<td>Stadium Capacity</td>
<td>restriction</td>
<td>restriction</td>
<td>restriction</td>
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</table>
TABLE 5.1-Continued

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>NFCE</td>
<td>Dummy for division NFC East</td>
<td>5.89</td>
<td>175.57</td>
<td>108.40</td>
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<td></td>
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<td>(1.37)</td>
<td>(-1.72)</td>
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<td>Dummy for division NFC North</td>
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<td>11.39</td>
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<td></td>
<td></td>
<td>(-0.16)</td>
<td>(0.13)</td>
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<td>NFCS</td>
<td>Dummy for division NFC South</td>
<td>-3.22</td>
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<td>(-0.75)</td>
<td>(-1.22)</td>
<td>(-1.64)</td>
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<td>Dummy for division NFC West</td>
<td>-5.39</td>
<td>-176.49</td>
<td>-188.25</td>
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<tr>
<td></td>
<td></td>
<td>(-1.28)</td>
<td>(-2.01)*</td>
<td>(-2.26)*</td>
</tr>
<tr>
<td>AFCE</td>
<td>Dummy for division AFC East</td>
<td>4.96</td>
<td>189.05</td>
<td>149.68</td>
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<td></td>
<td></td>
<td>(1.13)</td>
<td>(1.84)</td>
<td>(1.74)</td>
</tr>
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<td>AFCN</td>
<td>Dummy for division AFC North</td>
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<tr>
<td></td>
<td></td>
<td>(0.02)</td>
<td>(0.03)</td>
<td>-</td>
</tr>
<tr>
<td>AFCS</td>
<td>Dummy for division AFC South</td>
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<td>-94.87</td>
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<td></td>
<td></td>
<td>(-0.82)</td>
<td>(-1.11)</td>
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<td>YEAR03</td>
<td>Dummy for 2003 season</td>
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<tr>
<td></td>
<td></td>
<td>(-1.14)</td>
<td>(-1.98)*</td>
<td>(-2.06)*</td>
</tr>
<tr>
<td>YEAR04</td>
<td>Dummy for 2004 season</td>
<td>-2.07</td>
<td>-82.23</td>
<td>-99.97</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-0.86)</td>
<td>(-1.66)</td>
<td>(-1.79)</td>
</tr>
<tr>
<td>R-squared</td>
<td></td>
<td>0.28</td>
<td>0.42</td>
<td>0.48</td>
</tr>
<tr>
<td>Adjusted R-Squared</td>
<td></td>
<td>0.12</td>
<td>0.30</td>
<td>0.42</td>
</tr>
</tbody>
</table>

*Significant at a 5% significance level, (t-critical = 1.96)

Model 1 represents the original regression outlined in chapter 3. This initial model posed a few problems. First, the fit of the model was relatively low with only a 0.28 R-squared value and a 0.12 Adjusted R-squared value. This means Model 1 only explained 28% of the variation in the dependent variable PRICE. Another major problem was the lack of significant independent variables. Only attendance (ATTEND), alternative forms of recreation (ALTREC), and median income (WEALTH) were
significant at the 5% significance level in determining PRICE. The fact that previous research by Ferguson et al. (1991)¹ and Boyd and Boyd (2001)² found far more than these three variables significant in similar models questions the validity of Model 1. Therefore, adjustments were made in the other two models to improve both the fit and independent variable significance.

Model 2 incorporates these adjustments and drastically improves the estimation. In Model 2, PRICE is raised by an exponential factor of 1.76. This change, along with the corresponding alteration of the restriction equation, increased the R-squared to 0.42 and the Adjusted R-squared to 0.30. Model 2 therefore explains 42% of the variation in PRICE around its mean. Although this may sound a little low, it actually is a fairly good fit considering only three years of data and 93 observations were used. Even more important was the fact that six independent variables were significant at the 5% significance level.

Model 3 had the best fit of any model tested with a R-squared value of 0.48 and an Adjusted R-squared value of 0.42. This means the model explained just below half the variation in PRICE. Even though this model had the best fit, too many random variables were omitted from the estimation. Boyd and Boyd (2001)³ and Ferguson et al. (1991)⁴ studies both showed these variables to be important determinants of ticket prices

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³ Ibid.

and attendance. Having such a random selection of variables questions the validity of this model.

In the end, Model 2 was chosen as the best model for this study. Since the exclusion of relevant details is a larger econometric problem than the inclusion of irrelevant variables, this study, like Ferguson, Stewart, Jones, and Le Dressay (1991), decided to face the less serious problem by using the full set of variables. All variables used in the regression were well established by previous research. Therefore, it was appropriate to use a model that incorporated these variables. Even though Model 2 sacrificed a small drop in the R-squared value, it was still the most valid estimation of PRICE. Therefore, Model 2's coefficients were used for all numerical analysis, which is discussed later in the chapter.

It is important to note that the Ferguson et al. (1991) study stated that the significance, sign, and magnitude of independent variables are of very little interest and don't carry strong intuition with them. Since all the testable implications of the theory are in terms of the first and second order profit maximizing conditions the individual variable coefficients are not analyzed and only reported for completeness. This research has created a relatively good model for PRICE, which is important to accomplish before performing numerical analysis of the profit maximizing conditions. If the model for PRICE is not valid, then any results derived from the model are questionable. In the following section the results of Model 2 will be explained in greater detail with a focus on individual variables.

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5 Ibid.
6 Ibid.
Attendance Effects

This study found attendance (ATTEND) to have a negative impact on ticket prices. The partial regression coefficient is $-0.0023$ giving a t-statistic of $-2.07$. This means in order to have one more person attend the game the average ticket price must drop by 0.0023 dollars. This finding is consistent with Ferguson et al. (1991) research.\footnote{Ibid.} The fact that the partial coefficient was negative is central to the theory of profit maximization. The demand for attending an NFL game is a normal good such that as ticket prices go up less people are willing to pay to watch the game. The partial coefficient was also significant at the 5% significance level. This was important because the partial coefficient for ATTEND plays a major role in analyzing the first and second profit maximizing conditions. If this partial coefficient was not significant, the results of the numerical analysis would not be credible.

Player Expense Effects

As mentioned in chapter 4, player expenses (PEXP) was not found to be an endogenous variable in the regression, so it was deemed exogenous. This regression found PEXP to have a positive but small partial coefficient of $4.57E-07$ with a t-statistic of 0.19. This positive relationship with PRICE was as expected; however, the partial coefficient was insignificant. Therefore, the inclusion of PEXP in this model was not as necessary as originally thought. It's not certain why PEXP doesn't play a larger role in determining ticket price. Perhaps the fact that the NFL has a salary cap reduces the
significance of player expenses. The only thing that can be drawn from these results is that PEXP doesn’t play a significant role in the model.

**Demographic Effects**

Both population (POP) and median income (WEALTH) were used as the demographic variables when determining average ticket price. POP was found to have a highly significant relationship with ATTEND, leading to its use as the instrumental variable for ATTEND in the model. Since POP was used as an instrumental variable it wasn’t present as a direct determinant of PRICE. Therefore, no partial coefficients were estimated for it.

WEALTH was found to be significant and positively related to average ticket price. The partial coefficient was 0.0021 with a t-statistic of 5.68. This shows that as median income goes up by one dollar, average ticket price also increases by 0.0002 dollars. This result, which is consistent with the Ferguson et al. (1991) study, suggests that attending NFL games is a normal good.  

**Alternative Recreation Effects**

The number of different professional sports teams in an NFL team’s territory (ALTREC) was found to have a significant and positive relationship with PRICE. The partial coefficient for ALTREC was 77.21 with a t-statistic of 3.95. This was neither consistent with Boyd and Boyd’s (2001) research nor was it predicted in chapter 4.  

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8 Ibid.

9 Boyd and Boyd, “The Home Field Advantage: Implications for the Pricing of Tickets to Professional Team Sporting Events,” 254-64.
However, these results don’t come as a huge surprise. In this model, other professional sports teams were thought to compete as substitutes with the NFL franchises. As it turns out, these other teams don’t compete with NFL organizations, but instead increase the pride fans have for teams in their territory. Therefore, ALTREC is no longer thought of as a measure of substitutes. This research has shown that ALTREC actually increases support of a NFL team because people in the territory take pride in the teams representing them. This pride increases the demand for attendance and therefore owners are able to charge higher ticket prices.

Team Success Effects

The win percentage of an NFL team did play a role in determining ticket prices. Two winning percentages were used to analyze the effects of team success. The teams previous year’s winning percentage (PWIN) was significantly and positively related to average ticket price. PWIN had a partial coefficient of 364.43 with a t-statistic of 3.05. This means the more a team wins in the previous season, the higher ticket prices will become the following year. This is consistent with the research of Ferguson et al. (1991) and Boyd and Boyd (2001)\textsuperscript{11}. Therefore, this research showed prior team success does play a significant role in determining ticket prices.

The current season’s win percentage (CWIN) also had a positive relationship with PRICE but was found to be insignificant. The partial coefficient was 200.06 given a t-statistic of 1.36. This result was somewhat expected. An owner has to set ticket prices

\textsuperscript{10} Ferguson, "The Pricing of Sports Events: Do Teams Maximize Profit?" 297-310.

\textsuperscript{11} Boyd and Boyd, "The Home Field Advantage: Implications for the Pricing of Tickets to Professional Team Sporting Events," 254-64.
before the season begins. He doesn't know exactly how well his team will perform in the upcoming year and can only use a predicted value of success. Sometime this value is not accurate, which is most likely why CWIN was insignificant. Since the actual value of CWIN is not available to the owner at the time he makes his ticket pricing decision it is not surprising that CWIN does not play a significant role in Model 2.

**Star Power Effects**

The number of Pro Bowl players on an NFL team (STAR) had a negative relationship with ticket prices with a partial coefficient of $-0.11$. This result contradicts the expected positive relationship and the findings of Ferguson et al. (1991). However, the t-statistic for this partial coefficient is not significant at a value of $-0.01$. Such a low t-statistic means the validity of this partial coefficient is questionable. Therefore, the partial coefficient for STAR is not considered accurate in Model 2.

**Competitive Balance Effects**

As mentioned in the previous chapter, the Herfindahl Hirschman Index for competitive balance (HHI) was found to produce a near singular matrix. The production of a near singular matrix caused HHI to be dropped from all models.

\[ \text{Ferguson}, "\text{The Pricing of Sports Events: Do Teams Maximize Profit?}" 297-310. \]
Dummy Variables

Variables were used to account for the divisional effects throughout the NFL. Only the NFC Western Division (NFCW) showed significance in the regression with a partial coefficient of $-176.49$ and a t-statistic of $-2.01$. This means that every team in the NFCW should decrease ticket prices by $-176.50$ dollars below the intercept of Model 2 (890.99). Every other division's partial coefficient was insignificant at the 5% significance level and was not analyzed in detail.

The other set of dummy variables used in this data set accounted for the different seasons of the NFL. The 2003 season variable (YEAR03) was significant and negatively related to ticket prices with a partial coefficient of $-101.25$ and a t-statistic of $-1.98$. The 2004 season's variable (YEAR04) also had a negative relationship with price. It's partial coefficient of $-82.23$, although insignificant given a t-statistic of $-1.66$, was more positive than YEAR03. The 2005 season did not have a dummy variable, so it is assumed its partial coefficient value is 0, which is greater than YEAR04. This increasing trend shows that ticket prices have risen on average in every season.
Numerical Analysis of the Profit Maximization Conditions

Following the regression, the equations presented in FIGURE 5.1 were used to determine the first and second order derivatives for each observation (See APPENDIX B for derivation).

FIGURE 5.1
First and Second-Order Derivatives for Profit with Respect to Attendance

\[ \frac{\partial \pi}{\partial A} = A \cdot \beta_1 \cdot \left( \frac{1}{x} \right) \cdot p^{1-x} + P \]

\[ \frac{\partial^2 \pi}{\partial A^2} = \frac{2 \beta_1}{x} \cdot p^{1-x} + \frac{4 \beta_1^2 (1-x)}{x^2} \cdot p^{1-2x} \]

In these equations \( P \) is average ticket price, \( A \) is attendance, \( x \) is the exponent PRICE is raised to, and \( \beta_1 \) represents the partial coefficient of attendance. The values for the derivatives were calculated by plugging in the individual \( A \) and \( P \) values found in each observation’s data set and the \( \beta_1 \) and \( x \) values determined by Model 2’s STSLS regression. TABLE 5.2 shows the results of this analysis for every team in the NFL over all three seasons. The t-statistics are reported in parentheses below each derivative value (See APPENDIX C for t-stat derivation). These t-statistics test the null hypothesis that the derivative value is not equal to zero. If the t-statistic value is greater than the t-critical value of 1.96 for 5% significance level, then the derivative value is significantly greater than zero.
### TABLE 5.2

Numerical Results for the First and Second Derivatives of the Profit Function

<table>
<thead>
<tr>
<th>Team</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2004</th>
<th>2005</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\frac{\partial x}{\partial A}$</td>
<td>$\frac{\partial^2 x}{\partial A^2}$</td>
<td>$\frac{\partial x}{\partial A}$</td>
<td>$\frac{\partial^2 x}{\partial A^2}$</td>
<td>$\frac{\partial x}{\partial A}$</td>
</tr>
<tr>
<td>Arizona Cardinals</td>
<td>2.61</td>
<td>-0.0001999</td>
<td>2.66</td>
<td>-0.00020</td>
<td>8.28</td>
</tr>
<tr>
<td></td>
<td>(0.125)</td>
<td>(-0.0091)</td>
<td>(0.128)</td>
<td>(-0.0091)</td>
<td>(0.423)</td>
</tr>
<tr>
<td>Atlanta Falcons</td>
<td>19.46</td>
<td>-0.0001631</td>
<td>19.49</td>
<td>-0.000163</td>
<td>38.14</td>
</tr>
<tr>
<td></td>
<td>(1.200)</td>
<td>(-0.0124)</td>
<td>(1.202)</td>
<td>(-0.0124)</td>
<td>(2.815)</td>
</tr>
<tr>
<td>Baltimore Ravens</td>
<td>19.63</td>
<td>-0.000156</td>
<td>25.73</td>
<td>-0.0001439</td>
<td>29.22</td>
</tr>
<tr>
<td></td>
<td>(1.080)</td>
<td>(-0.0120)</td>
<td>(1.504)</td>
<td>(-0.0133)</td>
<td>(1.743)</td>
</tr>
<tr>
<td>Buffalo Bills</td>
<td>22.07</td>
<td>-0.000159</td>
<td>25.41</td>
<td>-0.0001513</td>
<td>26.20</td>
</tr>
<tr>
<td></td>
<td>(1.435)</td>
<td>(-0.0131)</td>
<td>(1.686)</td>
<td>(-0.0139)</td>
<td>(1.768)</td>
</tr>
<tr>
<td>Carolina Panthers</td>
<td>18.11</td>
<td>-0.0001617</td>
<td>18.01</td>
<td>-0.0001618</td>
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</tr>
<tr>
<td></td>
<td>(1.0336)</td>
<td>(-0.0119)</td>
<td>(1.024)</td>
<td>(-0.0118)</td>
<td>(1.211)</td>
</tr>
<tr>
<td>Chicago Bears</td>
<td>12.30</td>
<td>* -0.0001753</td>
<td>16.77</td>
<td>* -0.0001644</td>
<td>21.60</td>
</tr>
<tr>
<td></td>
<td>(0.664)</td>
<td>(-0.0108)</td>
<td>(0.943)</td>
<td>(-0.0116)</td>
<td>(1.274)</td>
</tr>
<tr>
<td>Cincinnati Bengals</td>
<td>14.87</td>
<td>* -0.0001806</td>
<td>24.87</td>
<td>* -0.0001557</td>
<td>32.70</td>
</tr>
<tr>
<td></td>
<td>(0.947)</td>
<td>(-0.0116)</td>
<td>(1.748)</td>
<td>(-0.0140)</td>
<td>(2.476)</td>
</tr>
<tr>
<td>Cleveland Browns</td>
<td>38.12</td>
<td>-0.0001328</td>
<td>-3.77</td>
<td>-0.0002271</td>
<td>-2.52</td>
</tr>
<tr>
<td></td>
<td>(3.192)</td>
<td>(-0.0185)</td>
<td>(-0.175)</td>
<td>(-0.0082)</td>
<td>(-0.122)</td>
</tr>
<tr>
<td>Dallas Cowboys</td>
<td>20.04</td>
<td>-0.0001527</td>
<td>32.00</td>
<td>-0.0001325</td>
<td>31.98</td>
</tr>
<tr>
<td></td>
<td>(1.082)</td>
<td>(-0.0121)</td>
<td>(1.922)</td>
<td>(-0.0146)</td>
<td>(1.920)</td>
</tr>
<tr>
<td>Denver Broncos</td>
<td>3.73</td>
<td>-0.0001962</td>
<td>7.65</td>
<td>-0.0001857</td>
<td>13.55</td>
</tr>
<tr>
<td></td>
<td>(0.180)</td>
<td>(-0.0093)</td>
<td>(0.387)</td>
<td>(-0.0099)</td>
<td>(0.730)</td>
</tr>
<tr>
<td>Detroit Lions</td>
<td>27.87</td>
<td>* -0.0001456</td>
<td>31.54</td>
<td>* -0.0001391</td>
<td>40.06</td>
</tr>
<tr>
<td></td>
<td>(1.855)</td>
<td>(-0.0144)</td>
<td>(2.172)</td>
<td>(-0.0153)</td>
<td>(2.976)</td>
</tr>
<tr>
<td>Green Bay Packers</td>
<td>47.76</td>
<td>* -0.0001157</td>
<td>47.63</td>
<td>* -0.0001148</td>
<td>66.88</td>
</tr>
<tr>
<td></td>
<td>(3.589)</td>
<td>(-0.0197)</td>
<td>(3.563)</td>
<td>(-0.0196)</td>
<td>(5.770)</td>
</tr>
<tr>
<td>Houston Texans</td>
<td>2.19</td>
<td>* -0.0002047</td>
<td>2.34</td>
<td>-0.000212</td>
<td>23.18</td>
</tr>
<tr>
<td></td>
<td>(0.108)</td>
<td>(-0.0091)</td>
<td>(0.121)</td>
<td>(-0.0091)</td>
<td>(1.707)</td>
</tr>
<tr>
<td>Indianapolis Colts</td>
<td>25.96</td>
<td>-0.0001463</td>
<td>31.85</td>
<td>-0.0001328</td>
<td>38.78</td>
</tr>
<tr>
<td></td>
<td>(1.484)</td>
<td>(-0.0133)</td>
<td>(1.910)</td>
<td>(-0.0145)</td>
<td>(2.469)</td>
</tr>
<tr>
<td>Jacksonville Jaguars</td>
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<td>-0.0001139</td>
<td>31.87</td>
<td>-0.0001324</td>
<td>38.69</td>
</tr>
<tr>
<td></td>
<td>(1.393)</td>
<td>(-0.0130)</td>
<td>(1.897)</td>
<td>(-0.0145)</td>
<td>(2.437)</td>
</tr>
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<td>Kansas City Chiefs</td>
<td>32.17</td>
<td>-0.0002124</td>
<td>34.25</td>
<td>-0.0001407</td>
<td>33.49</td>
</tr>
<tr>
<td></td>
<td>(2.494)</td>
<td>(-0.0164)</td>
<td>(2.878)</td>
<td>(-0.0175)</td>
<td>(2.730)</td>
</tr>
<tr>
<td>Miami Dolphins</td>
<td>32.98</td>
<td>-0.000145</td>
<td>30.30</td>
<td>-0.0001396</td>
<td>35.98</td>
</tr>
<tr>
<td></td>
<td>(2.202)</td>
<td>(-0.0155)</td>
<td>(1.985)</td>
<td>(-0.0148)</td>
<td>(2.475)</td>
</tr>
<tr>
<td>Minnesota Vikings</td>
<td>23.82</td>
<td>-0.0001425</td>
<td>21.90</td>
<td>-0.000158</td>
<td>28.50</td>
</tr>
<tr>
<td></td>
<td>(1.605)</td>
<td>(-0.0137)</td>
<td>(1.389)</td>
<td>(-0.0123)</td>
<td>(1.924)</td>
</tr>
<tr>
<td>New England Patriots</td>
<td>11.25</td>
<td>-0.0001427</td>
<td>11.73</td>
<td>-0.0001857</td>
<td>20.85</td>
</tr>
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<td>(0.655)</td>
<td>(-0.0107)</td>
<td>(0.692)</td>
<td>(-0.0108)</td>
<td>(1.271)</td>
</tr>
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<td>New Orleans Saints</td>
<td>24.79</td>
<td>-0.0001296</td>
<td>34.46</td>
<td>-0.000134</td>
<td>34.17</td>
</tr>
<tr>
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<td>(1.547)</td>
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<td>(2.417)</td>
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<td>(2.373)</td>
</tr>
<tr>
<td>New York Jets</td>
<td>3.61</td>
<td>-0.0001877</td>
<td>1.52</td>
<td>-0.0002115</td>
<td>4.94</td>
</tr>
<tr>
<td></td>
<td>(0.189)</td>
<td>(-0.0093)</td>
<td>(0.076)</td>
<td>(-0.0089)</td>
<td>(0.256)</td>
</tr>
</tbody>
</table>
The values reported in TABLE 5.2 were used to test the following conditions of profit maximization shown in FIGURE 5.2 (See APPENDIX B).
FIGURE 5.2

Conditions of Profit Maximization

- First-Order Profit Maximization Condition
  
i) \( \frac{\partial \pi}{\partial A} = 0 \) for non-sellout team
  
ii) \( \frac{\partial \pi}{\partial A} \geq 0 \) for sellout teams

- Second-Order Profit Maximization Condition
  
i) \( \frac{\partial \pi^2}{\partial A^2} < 0 \) for non-sellout teams
  
ii) \( \frac{\partial \pi^2}{\partial A^2} \) unrestricted for sellout teams

The following two sections will discuss how the results presented in TABLE 5.2 relate to the profit maximization conditions.

First-Order Conditions

The first-order conditions of profit maximization are very strongly supported by the estimation results provide in TABLE 5.2. Numerical evaluation showed that all sellout teams had a first derivative that was greater than or equal to zero. This means every single sellout team satisfied the first-order condition necessary for profit maximization. 11 out of 28 sellout teams had a positive first derivative that was significantly greater than zero at the 5% significance level. The other 17 sellout teams all had positive first derivatives that were not significantly different from zero.

The fact that every sellout team had a positive derivative that satisfied the profit maximization condition questions the biasness of the t-statistic test. If the t-statistic test
was biased, this test of the profit maximizing conditions would not be valid. However, these suspicions are dispelled by the fact that non-sellout teams’ first derivatives take on both positive and negative values. Also, some of these second derivatives fail to accept the profit maximization conditions.

Non-sellout teams tend to support the first-order profit maximization condition but with some mixed results. The first derivative should not differ significantly from zero for non-sellout teams, but in fact 17 out of the 68 non-sellout teams actually have a significantly positive first derivative at the 5% significance level. This occurred for seven teams in a single season, San Francisco in two season, and Oakland and Philadelphia in all three seasons. According to the first derivative, these teams are not setting ticket prices at a profit maximizing level and need to raise prices if they wish to do so. In the other 51 non-sellout seasons the first derivative wasn’t significantly different from zero, and therefore satisfied the first-order condition. This means 75% of the non-sellout teams satisfy the first order conditions necessary for profit maximization.

Of the 96 observations tested in the NFL during the 2003 through the 2005 season 79 out of 96 teams, or 82%, produce first derivatives consistent with profit maximization’s first-order conditions. This shows that a very high percentage of the first-order results are in accordance with the conditions outlined for the model.

**Second-Order Conditions**

The results associated with the second-order profit maximization condition produced somewhat mixed results. The second-order condition states that all non-sellout teams must have a second derivative that is less than zero, and all sellout teams’ second
derivative is unrestricted. Numerical evaluation showed every second derivative having a negative value that was not significantly different from zero. While these values are completely acceptable for all sellout teams based on the second-order condition, they are unacceptable for non-sellout teams. Although the second derivatives are negative, their insignificance questions their credibility.

The second-order condition of profit maximization determines whether a certain price level is profit maximizing or minimizing. If the value is negative, the price level is closer to the profit maximizing position. If it is positive, the price level is closer to a profit minimizing position. Even though the second derivatives are not significantly different from zero, the fact that they all have negative values supports the theory that they are closer to a profit maximizing position. Also, given that research by Forbes found all NFL franchises to be making a positive profit, it is hard to believe that any NFL franchise would be operating at a profit minimizing position. Therefore, this study concludes all teams are setting ticket prices at a level consistent with profit maximization, not profit minimization. Even though the second-order derivatives do not fully support this statement, other sources of information overcome the non-supportive second derivative results.

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Conclusions

This study has attempted to determine if NFL franchises act as profit maximizing businesses by setting ticket prices at a level corresponding the profit maximization conditions. The theory of profit maximization has been presented in literature since the beginning of the 20th century. Profit maximization occurs when a business determines the quantity of a good that returns the largest possible profit. Professional sports franchises are by definition a business, but are they focused on making the most profit possible? This study addresses the profit theory mentioned in the previous question by focusing on the gate receipts generated by ticket prices and attendance at NFL football games.

Many studies have addressed the determinants of price and attendance in professional sports leagues including the NFL. These studies all find similar results which are for the most part confirmed by this study. There have also been a handful of other studies that focused on the profit maximization in other industries. However, only one other study has ever attempted to examine the profit maximization theory in combination with a professional sports league. This research combined the results found in previous studies addressing the determinants of price and attendance with the Ferguson et al. (1991) study to test the theory of profit maximization in the NFL.

Team information and data was collected for the 2003 through the 2005 NFL seasons in order to find a suitable regression for the determinants of ticket price. Every team during each season was used as an observation giving a total of 96 data points.

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14 Frank Hyneman Knight, "Risk, Uncertainty and Profit," (1921).

STSLS regression analysis was then used to estimate the determinants of ticket prices in the NFL given the cross equation restriction involving the ceiling placed on attendance due to the capacity of a team’s stadium. This cross equation restriction was determined by implementing a univariate Kuhn-Tucker approach with regards to capacity constraints in the NFL. The results of this regression, analyzed previously in this chapter, were consistent with previous studies for the most part. Only alternative forms of recreation and star power had partial coefficient relationships opposite to those found in previous studies. However, the partial coefficient for star power was not significant, and positive relationship alternative recreation had with ticket price was presumably accounted for through the “pride factor” territories have in their teams.

After the STSLS regression was acceptable, numerical analysis was conducted to test the first and second order conditions of profit maximization under Kuhn-Tucker constraints. These numerical evaluations combined individual observation data and values obtained from the STSLS regression in order to produce the first and second derivatives of the profit function with respect to attendance. The results produced through the above analysis offered strong support for the hypothesis that NFL franchises act in accordance with profit maximizing behavior. This evidence suggests that behavior of professional sport franchises may reflect the same profit maximizing approach seen in other competitive market industries. The findings of this research provide a potential gateway towards microeconomic research in the area of professional sports. The following section includes various extensions of this research that could possibly be implemented in future studies.
Future Research

The first improvement to this study would be to increase the number of observations in the data set. Observing more than three years of data could possibly improve the STSLS regression, producing an even more reliable model for the numerical analysis. If enough years were made available, a dynamic profit maximization model could be constructed to analyze profit maximization over a length of time instead of a single season. Dynamic analysis could extend on the model created in this study, and evaluate the profit maximization decisions made by franchises in the long run.

The next major step in constructing this dynamic profit maximization model for a professional sports market is to incorporate franchise values. Examining profit from a yearly operating revenue standpoint only helps owners maximize earnings in a given time period. Owners do not mind spending money in the short run if it will increase their profits in the long run. Oftentimes owners may break even in a given time period while the resale value of their franchise increases at a healthy rate. The owner views the team as an asset, and while he may spend much of the team’s earnings in a given year, he could still be profit maximizing in the long run. An example of this dynamic theory is the New England Patriots 2004 and 2005 seasons. In 2004 New England had an operating income, which is representative of yearly profit, of $50.5 million and a current franchise value of $1,040 million.\(^{16}\) In 2005 operating income was $43.6 million and the current franchise value was $1,176 million.\(^{17}\) From these statistics it is easy to see that yearly operating profit actually fell by 14% but the resale value of the franchise increased

\(^{16}\) "NFL Team Valuations."

\(^{17}\) Ibid.
by 13%. However, an increase of 13% percent from $1,040 million dollars is much larger than a fall of 14% from $50.5 million. In the end, the owners lost $6.9 million in yearly operating profit but gained $136 million in resale value, resulting in a net profit rise of $129.1 million in a single year. This basic example stresses the importance of a dynamic model. A dynamic model could incorporate both operating revenue and franchise value to test if owners are truly profit maximizing over a length of time.

Another major improvement could be to expand the model to include all forms of revenue and costs that vary between teams. Additional variable revenue and cost data could not be found for this study. Therefore, this model used a simplified expression for the profit function using gate revenue and player expenses. Other forms of variable revenue include local television deals, advertisement/sponsor contracts, and stadium concessions. Additional variable costs included such things as stadium contracts and operating expenses. With these additional measures, a regression including attendance as a determinant could be estimated for both variable revenue and variable cost. Using these regressions a new set of derivatives could be constructed and the profit maximization conditions could again be tested. Extended research that devised a model that incorporated all these variable measurements could produce results that more accurately examine profit maximizing behavior in the NFL.

One more avenue for research could be to examine different types of game ‘viewership’ and the respective prices they entail. All stadiums classify different groups of seats with various prices. The higher priced seats, such as luxury suites and club level seating, are supposed to produce a more enjoyable game day attendance. Research could
examine the pricing of all the different types of seating at games and how they individually affect profit maximization.

While this study examined only the effects of variables in a given year, there are many variables that could potentially have lagged effects on profit. One example is star power. Say a player is selected into the Pro Bowl in year $t$. His contract might be renegotiated over the off-season, leading to a higher salary for him and greater player expenses for the team in year $t+1$. To counter the increased player expenses, ticket prices will also have to rise in year $t+1$ and maybe even year $t+2$. Testing the lagged effects of variables on ticket prices and profit is definitely a potential avenue for future research.

Lastly, this research, and the future extensions mentioned above, could also motivate similar research in other professional sports leagues. Are ticket prices in other leagues such as the National Basketball Association and Major League Baseball consistent with profit maximizing behavior?

Implications

This study offers some important implications in terms of the operation of NFL franchises. The model produced in this study could serve as the basis for determining what ticket prices return the most profit for individual NFL franchises. Could a team return a greater profit if it charged higher ticket prices? The analysis provided in this research could forecast whether an NFL franchise should raise or lower ticket prices in order to return the greatest profit.

By no means is this model perfect, but it does represent a starting point. The NFL is an extremely successful professional sports league, with every single team earning a
large profit. The results of this study clearly show that a large amount of these NFL teams act in agreement with profit maximizing behavior each season, with 79 out of the 96 observations significantly supporting the profit maximization conditions. However, due to inevitable changes in teams, territories, and the league, the ticket prices charged throughout the NFL will constantly be changing. The information provided by this research could help all NFL franchises prosper in the future through ticket price setting at profit maximizing levels.

In conclusion, this research provides a strong base for examining profit maximization decisions in the NFL. The study found that 82% of the ticket prices in the NFL corresponded with profit maximizing behavior. If the profit function could be redefined to incorporate all variable revenue and cost, as mentioned above, more significant and beneficial results could possibly be obtained. The results of this study are intriguing. They go against the idea that franchises always try to win the most games as possible. Winning, although related to profit, is definitely not the only thing on the mind of NFL owners. Perhaps professional sports leagues display more microeconomic characteristics seen in other competitive industries than previously thought.

\[18\textsuperscript{18} \text{Ibid.}\]
The purpose of this appendix is to outline the Kuhn-Tucker conditions presented in Chapter 3. The Kuhn-Tucker conditions were used because there was a ceiling restriction placed on the attendance variable of our model. Since this constraint existed, an interior solution couldn’t be assumed for the profit maximization conditions. Therefore, the Kuhn-Tucker conditions were used to incorporate the appropriate restriction equation into the model.

A univariate Kuhn-Tucker approach was used because only one variable, attendance, was restricted in the model due to each stadium’s capacity constraint. The basic model, presented in equation A.1, shows this restriction,

\[ \text{Max } \pi \text{ subject to } A \leq C \]  \hspace{1cm} \text{(A.1)}

, where \( \pi \) represents profit, \( A \) represents attendance, and \( C \) represents capacity. Since profit is some function of attendance the problem posed in this study was as follows.

\[ \text{Max } f(A) \text{ subject to } A \leq C \]  \hspace{1cm} \text{(A.2)}

The solution to this problem can be captured in three unique situations: internal, ceiling, and combination.
Internal: $f'(A^*)=0$ and $A^*<C$

Ceiling: $f'(A^*)>0$ and $A^*=C$

Combination: $f'(A^*)=0$ and $A^*=C$
By combining the necessary conditions for internal, external, and combination the following set of restrictions are achieved.

\[ f'(A^*) \geq 0; \quad C - A \geq 0; \quad f'(A^*) \times (C - A) = 0 \quad \text{A.3} \]

Equation A.3 respectively states that in every situation the derivative of the profit function with respect to attendance can’t have a negative value, difference between attendance and capacity is always greater than or equal to zero, and at least one, if not both, of the previous conditions must have a value of zero.

The third part of equation A.3 was used as the restriction equation in this study because it is all encompassing. The restriction equation can be seen in equation A.4.

\[ \frac{\partial \pi}{\partial A} \times (C - A) = 0 \quad \text{A.4} \]

Therefore, in order to satisfy the profit maximization conditions, if a team has a sellout season then \( C - A \) has to equal zero and \( f'(A^*) \) must be greater than or equal to zero. On the other hand, if a team does not sellout then \( f'(A^*) \) must be equal to zero and \( C - A \) is greater than zero. Using the restriction equation and the appropriate Kuhn-Tucker conditions, the profit maximization conditions were tested in Chapter 5.
APPENDIX B

Derivation of the Profit Maximization Conditions

The purpose of this appendix is to provide a detailed mathematical solution for the expressions for the first and second order derivatives of the profit function. These derivatives obtained are used to test the first and second order profit maximizing conditions. The model developed in the chapter 3 is a simplified version of the following model where $x=1$. The following equations outline the process of deriving the profit maximizing conditions, when price may be raised to a power $x$.

This goal of this model is to maximize profit, $\pi$, given that attendance, $A$, cannot exceed the capacity of a stadium, $C$. Equation B.1 represents this problem,

$$\text{Max } \pi = p^x \cdot A - E \quad \text{subject to } A \leq C$$

Equation B.1

where $p$ is the average ticket price and $E$ represents player expenses. Ticket prices can be represented by the following equation,

$$p^x = f(A, E, z; \theta)$$

Equation B.2

where $z$ is the vector of exogenous attributes of the team and $\theta$ is the vector parameters. Therefore, the inverse demand function from equation B.2 can be represented by equation B.3,

$$p^x = \beta_0 + \beta_1 A + \beta_2 E + \beta_k z$$

Equation B.3
where all the $\beta$’s represents the coefficients produced by the System Two-stage Least Squares regression. Also, $k$ represents the number of attributes in vector $z$. In order to isolate $p$, both side of equation B.3 are raised to the power of $\frac{1}{x}$, which can be seen in equation B.4.

$$p = \left( \beta_0 + \beta_1 A + \beta_2 E + \beta_k z \right)^{\frac{1}{x}}$$  \hspace{1cm} \text{B.4}

Now the right-hand side of equation B.4 can be substituted for $p$ in equation B.1 to produce equation B.5. Since profit maximization corresponds to revenue minus costs, equation B.5 will be the basis for all profit maximizing testing.

$$\pi = A^* \left( \beta_0 + \beta_1 A + \beta_2 E + \beta_k z \right)^{\frac{1}{x}} - E$$  \hspace{1cm} \text{B.5}

In order to derive the first profit maximizing condition the first-order derivative of B.5 must be taken and set equal to zero. The first order derivative of B.5 is achieved by using the product rule, the result of which can be seen in equation B.6.

$$\frac{\partial \pi}{\partial A} = A^* \left( \frac{1}{x} \right)^* \left( \beta_0 + \beta_1 A + \beta_2 E + \beta_k z \right)^{\frac{1-x}{x}}$$  \hspace{1cm} \text{B.6}

$$+ \left( \beta_0 + \beta_1 A + \beta_2 E + \beta_k z \right)^{\frac{1}{x}} = 0$$

Referring back to equation B.3, $P^*$ can be substituted into equation B.6 for

$$\beta_0 + \beta_1 A + \beta_2 E + \beta_k z$$, producing equation B.7.

$$\frac{\partial \pi}{\partial A} = A^* \left( \frac{1}{x} \right)^* \left( P^* \right)^{\frac{1-x}{x}} + \left( P^* \right)^{\frac{1}{x}} = 0$$  \hspace{1cm} \text{B.7}

Simplifying the exponents produces equation B.8.

$$\frac{\partial \pi}{\partial A} = A^* \left( \frac{1}{x} \right)^* P^{1-x} + P = 0$$  \hspace{1cm} \text{B.8}
Rearranging produces the first profit maximizing condition that is used in the numerical analysis of data, which is presented in equation B.9.

$$\frac{\partial \pi}{\partial A} = \frac{A^* \beta_1}{x} * P^{1-x} + P = 0$$  \hspace{1cm} \text{B.9}$$

Next, the second order derivative must be solved for in order to test the second condition of profit maximization. The condition states that the second-order derivative must be negative to be profit maximizing. The derivative of $$\frac{\partial \pi}{\partial A}$$ must be taken with respect to attendance (A). The starting point for calculating this second-order derivative is equation B.6. Equation B.6 is then simplified to form equation B.10.

$$\frac{\partial \pi}{\partial A} = \frac{A^* \beta_1}{x} \left( \beta_0 + \beta_1 A + \beta_2 E + \beta_k z \right)^{\frac{1-x}{x}} + \left( \beta_0 + \beta_1 A + \beta_2 E + \beta_k z \right)^{\frac{1}{x}}$$  \hspace{1cm} \text{B.10}$$

Next, the derivative of B.10 is taken with respect to A through the use of the product rule. The result is shown in equation B.11

$$\frac{\partial^2 \pi}{\partial A^2} = \frac{\beta_1}{x} \left( \beta_0 + \beta_1 A + \beta_2 E + \beta_k z \right)^{\frac{1-x}{x}}$$

$$+ \frac{A^* \beta_1}{x} \left( \beta_0 + \beta_1 A + \beta_2 E + \beta_k z \right)^{\frac{1-x}{x}}$$

$$+ \frac{\beta_1}{x} \left( \beta_0 + \beta_1 A + \beta_2 E + \beta_k z \right)^{\frac{1-x}{x}}$$  \hspace{1cm} \text{B.11}$$

Using equation B.3, $$P^x$$ can be substituted into equation B.11 for $$\beta_0 + \beta_1 A + \beta_2 E + \beta_k z$$, producing equation B.12.

$$\frac{\partial^2 \pi}{\partial A^2} = \frac{\beta_1}{x} \left( P^x \right)^{\frac{1-x}{x}} + \frac{A^* \beta_1}{x} \left( P^x \right)^{\frac{1-x}{x}} + \frac{\beta_1}{x} \left( P^x \right)^{\frac{1-x}{x}}$$  \hspace{1cm} \text{B.12}$$

Next, equation B.12 is simplified by combining the exponents to produce equation B.13
\[
\frac{\partial \pi^2}{\partial A^2} = \frac{\beta_1 \cdot p^{1-x} + A \beta_1^2 (1-x) \cdot p^{1-2x}}{x^2} + \frac{\beta_1 \cdot p^{1-x}}{x} \quad \text{B.13}
\]

Finally, the like terms can be combined to produce the second profit maximizing condition represented in equation B.14.

\[
\frac{\partial \pi^2}{\partial A^2} = 2 \frac{\beta_1 \cdot p^{1-x} + A \beta_1^2 (1-x) \cdot p^{1-2x}}{x^2} \quad \text{B.14}
\]

The results of this appendix, in combination with the Kuhn-Tucker conditions outlined in Appendix A are the following:

**First Profit Maximizing Condition**

- **Internal Solution:** \( \frac{A \cdot \beta_1 \cdot p^{1-x}}{x} + P = 0 \) for all non sellout teams
- **Ceiling Solution:** \( \frac{A \cdot \beta_1 \cdot p^{1-x}}{x} + P \geq 0 \) for all sellout teams

**Second Profit Maximizing Condition**

- \( \frac{2 \beta_1 \cdot p^{1-x} + A \beta_1^2 (1-x) \cdot p^{1-2x}}{x^2} < 0 \) for all non sellout teams
- \( \frac{2 \beta_1 \cdot p^{1-x} + A \beta_1^2 (1-x) \cdot p^{1-2x}}{x^2} \) is unrestricted for sellout teams
The purpose of this appendix is to provide a detailed solution of the t-statistics developed to test the significance of the numerical values of the profit function’s first and second-order derivatives in Chapter 5. These t-statistics test the null hypothesis that the numerical values calculated are equal to zero. If the t-statistics are above the critical value of 1.96 then the numerical values are significantly different than zero at the 95% confidence level. The derivation of these t-statistics are outlined below. The concept for the following solution is found in Goldberger (1964).

\[ t = \frac{\text{Value of } X_t - \mu}{SE_t} \]

**First-Order Profit Maximizing Condition (FPMC) t-statistic**

When testing the first profit maximizing condition,

\[ FPMC = \frac{\partial \pi}{\partial A} = A^* \beta_1 \left( \frac{1}{x} \right)^* P^{1-x} + P \]

where $\hat{\beta}_1$ is a random variable and $A$, $P$, and $x$ are not random. $\hat{\beta}_1$ is the partial coefficient of attendance and $x$ is the exponential power. Both are obtained from the STSLS regression. Attendance, $A$, and average ticket price, $P$, are the numerical values obtained from an observation of data. Since the value of $FPMC$ can easily be calculated by plugging in these values, the focus needs to be on solving for the standard error of $FPMC$, $SE_x$. To begin, the $Var(FPMC)$ is presented in equation C.2.

$$Var(FPMC) = Var\left(A \times \hat{\beta}_1 \times \left(\frac{1}{x}\right) \times P^{1-x} + P\right)$$  \hspace{1cm} C.2$$

Next, the right hand side of the equation is solved out. Using the fact that $Var(a_0Y + a_1) = a_0^2Var(Y)$ when $a_0$ & $a_1$ are non random and $Y$ is a random variable, equation C.3 can be produced.

$$Var(FPMC) = \left(\frac{AP^{1-x}}{x}\right)^2 Var(\hat{\beta}_1)$$  \hspace{1cm} C.3$$

Using the relationship $Var(Y) = SE_Y^2$, equation C.4 can be derived from equation C.3.

$$Var(FPMC) = \left(\frac{AP^{1-x}}{x}\right)^2 SE_{\hat{\beta}_1}^2$$  \hspace{1cm} C.4$$

Using the same relationship from above, equation C.5 is also valid.

$$Var(FPMC) = SE_{FPMC}^2 = \left(\frac{AP^{1-x}}{x}\right)^2 SE_{\hat{\beta}_1}^2$$  \hspace{1cm} C.5$$

Using the right hand side of equation C.5 the square root of each side can be taken to produce equation C.6.

$$\sqrt{SE_{FPMC}^2} = \sqrt{\left(\frac{AP^{1-x}}{x}\right)^2 SE_{\hat{\beta}_1}^2}$$  \hspace{1cm} C.6$$
Simplifying produces equation C.7.

\[ SE_{FPMC} = \frac{AP^{1-x}}{x} \times SE_{\hat{\beta}_1} \]  

C.7

Now that \( SE_{FPMC} \) has been calculated it can be substituted into the t-statistic formula. Therefore, the t-statistic for the first condition of profit maximization is presented in equation C.8.

\[ FPMC\ t-stat = \frac{Value\ of\ FPMC}{SE_{FPMC}} = \frac{A \times \hat{\beta}_1 \times \left(\frac{1}{x}\right) \times P^{1-x} + P}{\frac{AP^{1-x}}{x} \times SE_{\hat{\beta}_1}} \]  

C.8

Since \( A \) and \( P \) are both given data values, and \( \hat{\beta}_1 \), \( x \), and \( SE_{\hat{\beta}_1} \) can all be found through STSLS regression the \( FPMC\ t-stat \) can easily be calculated for each observation.

**Second-Order Profit Maximizing Condition (SPMC) t-statistic**

Given equation C.9,

\[ SPMC = \frac{\partial \pi^2}{\partial A^2} = \frac{2 \hat{\beta}_1}{x} \times (1-x) + \frac{A \hat{\beta}_1^2}{x^2} \times P^{1-2x} \]  

C.9

, where \( \hat{\beta}_1 \) and \( \hat{\beta}_1^2 \) are random variables and \( A, P, \) and \( x \) are not random. Again \( \hat{\beta}_1 \), \( \hat{\beta}_1^2 \), and \( x \) can be obtained through STSLS regression. \( A \) and \( P \) are values of a particular observation’s data set. Using equation C.9 the variance for the SPMC can be expressed as equation C.10.
\[ Var(\text{SPMC}) = Var\left(\frac{2\beta_1 \cdot p^{1-x} + A \beta_1^2 (1-x) \cdot p^{1-2x}}{x^2}\right) \] 

Given the relationship that \( Var(a_0 Y + a_1 Z) = a_0^2 Var(Y) + a_0 a_1 Cov(Y, Z) + a_1 Var(Z) \) when \( Y, Z \) are random variable and \( a_0, a_1 \) are not random, equation C.10 expands to equation C.11.

\[
Var(\text{SPMC}) = \left(\frac{2p^{1-x}}{x}\right)^2 Var(\hat{\beta}_1) + \left(\frac{2p^{1-x}}{x} \cdot A(1-x)p^{1-2x}/x^2\right) \cdot Cov(\hat{\beta}_1, \hat{\beta}_1^2) \\
+ \left(\frac{A(1-x)p^{1-2x}}{x^2}\right)^2 Var(\hat{\beta}_1^2)
\]

To simplify equation C.11 it is important to note that \( \hat{\beta}_1 \) and \( \hat{\beta}_1^2 \) have a nonlinear relationship. Therefore, since covariance only based on linear relationships,

\[
\text{Cov}(\hat{\beta}_1, \hat{\beta}_1^2) = 0.
\]

This means the entire middle term of equation C.11 drops out. This produces equation C.12.

\[
Var(\text{SPMC}) = \left(\frac{2p^{1-x}}{x}\right)^2 Var(\hat{\beta}_1) + \left(\frac{A(1-x)p^{1-2x}}{x^2}\right)^2 Var(\hat{\beta}_1^2)
\]

Using the relationship \( Var(Y) = SE_Y^2 \), equation C.13 can be derived from equation C.12

\[
Var(\text{SPMC}) = SE_{SPMC}^2 = \left(\frac{2p^{1-x}}{x}\right)^2 SE_{\hat{\beta}_1}^2 + \left(\frac{A(1-x)p^{1-2x}}{x^2}\right)^2 SE_{\hat{\beta}_1^2}^2
\]

Taking the square root of the last two expressions in equation C.13 produces equation C.14.

\[
\sqrt{SE_{SPMC}^2} = \sqrt{\left(\frac{2p^{1-x}}{x}\right)^2 SE_{\hat{\beta}_1}^2 + \left(\frac{A(1-x)p^{1-2x}}{x^2}\right)^2 SE_{\hat{\beta}_1^2}^2}
\]
Simplifying gives equation C.15.

\[
SE_{SPMC} = \sqrt{\left(\frac{2P^{1-x}}{x}\right)^2 SE_{\beta_1}^2 + \left(\frac{A(1-x)P^{1-2x}}{x^2}\right)^2 SE_{\beta_1}^2} \tag{C.15}
\]

Now that \(SE_{SPMC}\) has been calculated it can be substituted into the t-statistic formula. Therefore, the t-statistic for the second condition of profit maximization is presented in equation C.16.

\[
SPMC\ t-stat = \frac{Value\ of\ SPMC}{SE_{SPMC}} = \frac{2 \hat{\beta}_1 * P^{1-x} + A \hat{\beta}_1^2 (1-x) * P^{1-2x}}{\sqrt{\left(\frac{2P^{1-x}}{x}\right)^2 SE_{\beta_1}^2 + \left(\frac{A(1-x)P^{1-2x}}{x^2}\right)^2 SE_{\beta_1}^2}} \tag{C.16}
\]

\(A\) and \(P\) are both given values, and \(\hat{\beta}_1, \hat{\beta}_1^2, SE_{\beta_1},\) and \(x\) can all be found through the STSLS regression. However, since \(\hat{\beta}_1^2\) is a nonlinear function of \(\hat{\beta}_1\), the value of \(SE_{\beta_1}\) must somehow be estimated using \(SE_{\beta_1^2}\). The following section outlines the process of this estimation.

\[SE_{\hat{\beta}_1^2} \] Estimation

In order to calculate \(SE_{\hat{\beta}_1^2}\) the Delta Method was used. The concept of using the Delta Method to approximate \(SE_{\hat{\beta}_1^2}\) was found in Davidson and MacKinnon (2004).\(^2\) The

Delta Method is a popular method of estimating the standard error of a nonlinear function through the use of asymptotic approximation. This study is interested in calculating the standard error of some parameter \( r \), where \( r \) is the monotonic nonlinear function of \( \beta \), shown in equation C.17.

\[
\gamma = g(\beta) = \beta^2 \tag{C.17}
\]

The way to approximate \( r \) is to use \( \hat{r} \) as it is presented in equation C.18.

\[
\hat{r} = g(\hat{\beta}) = \hat{\beta}_1^2 \tag{C.18}
\]

Since \( \hat{r} \) is a function of \( \hat{\beta}_1 \), then \( Var(\hat{r}) \) should be some function of \( Var(\hat{\beta}_1) \). The idea behind the Delta Method is to find a linear approximation of \( g(\beta) \) and then apply the fact that \( Var(a_0Y + a_1Z) = a_0^2Var(Y) + a_0a_1Cov(Y,Z) + a_1Var(Z) \) when \( Y, Z \) are random variable and \( a_0, a_1 \) are not random.

The most common way to obtain a linear approximation of a nonlinear function is to use Taylor's Theorem. Since this study only has one variable, a first-order Taylor expansion is used. The first-order Taylor expansion states

\[
f(a + h) \approx f(a) + hf'(a) \tag{C.19}
\]

, where \( a \) is the actual value of a parameter, and \( h \) is the difference between the estimated and actual value of the parameter \( (\hat{a} - a) \). Applying this first-order Taylor expansion to equation C.17 produces the following.

\[
g(\beta + (\hat{\beta}_1 - \beta)) \approx g(\beta) + (\hat{\beta}_1 - \beta)g'(\beta) \tag{C.20}
\]

Simplifying and substituting \( \hat{r} \) for \( g(\hat{\beta}_1) \) (from equation C.18) produces C.21.

\[
\hat{r} \approx g(\beta) + (\hat{\beta}_1 - \beta)g'(\beta) \tag{C.21}
\]

\[
\hat{r} \approx g(\beta) + (\hat{\beta}_1 - \beta)g'(\beta) \tag{C.21}
\]
Now the variance of \( \hat{y} \) can be taken, where \( \hat{\beta}_1 \) is a random variable and \( \beta \) is not random. The variance of \( \hat{y} \) is represented by equation C.22.

\[
\text{Var}(\hat{y}) \equiv \text{Var}(g(\beta) + (\hat{\beta}_1 - \beta)g'(\beta))
\]

C.22

Expanding produces equation C.23

\[
\text{Var}(\hat{y}) \equiv \text{Var}(g(\beta) + \hat{\beta}_1 * g'(\beta) - \beta * g'(\beta))
\]

C.23

Applying the relationship that \( \text{Var}(a_Y + a_i) = a_o^2 \text{Var}(Y) \) when \( a_o \) & \( a_i \) are non random and \( Y \) is a random variable, C.23 expands to produce equation C.24.

\[
\text{Var}(\hat{y}) \equiv g'(\beta)^2 \text{Var}(\hat{\beta}_1)
\]

C.24

Next, the relationship \( \text{Var}(Y) = SE_Y^2 \) can be applied to equation C.24 to produce equation C.25.

\[
SE_Y^2 = \text{Var}(\hat{y}) \equiv g'(\beta)^2 \text{Var}(\hat{\beta}_1)
\]

C.25

Taking the square root of equation C.25 produces the following.

\[
SE_{\hat{y}} = \sqrt{g'(\beta)^2 \text{Var}(\hat{\beta}_1)}
\]

C.26

Simplifying produces equation C.27.

\[
SE_{\hat{y}} \equiv |g'(\beta)|SE_{\hat{\beta}}
\]

C.27

Since \( g'(\beta) \) is an actual value, \( g'(\hat{\beta}_1) \) is used to approximate it. Substituting \( g'(\hat{\beta}) \) into equation C.27 produces equation C.28.

\[
SE_{\hat{y}} \equiv |g'(\hat{\beta}_1)|SE_{\hat{\beta}}
\]

C.28
Equation C.26 is confirmed in Davidson and McKinnon. Using equation C.18, $\hat{\beta}^2$ can be substituted for $\hat{\gamma}$ in equation C.28. $2\hat{\beta_1}$ can also be substituted into equation C.28 for $g'(\hat{\beta_1})$. These two substitutions produce equation C.29.

$$SE_{\hat{\beta}^2} \equiv |2\hat{\beta_1}|SE_{\hat{\beta_1}}$$

Since $\hat{\beta_1}$ and $SE_{\hat{\beta_1}}$ are found through STSLS regression, $SE_{\hat{\beta}^2}$ can now be solved for. Therefore, the $SPMC$ $t$-statistic presented in equation C.16 can be numerically evaluated.

The final formula for calculating the $SPMC$ $t$-statistic is presented in equation C.30.

$$SPMC \ t - \text{stat} = \frac{\text{Value of } SPMC}{SE_{SPMC}}$$

$$= \frac{2\hat{\beta_1} \cdot P^{1-x} + \frac{A\hat{\beta_1}^2 (1-x)}{x} \cdot P^{1-2x}}{\sqrt{\left(\frac{2P^{1-x}}{x}\right)^2 SE_{\hat{\beta_1}}^2 + \left(\frac{A(1-x)P^{1-2x}}{x^2}\right)^2 \left(2\hat{\beta_1}|SE_{\hat{\beta_1}}\right)^2}}$$

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APPENDIX D

EViews Code for System Two-Stage Least Squares Regression

The purpose of this appendix is to show the codes entered for this study’s three models. First, a system had to be created in EViews. To do this “New Object” was selected from the drop down “Objects” tab. Then “System” was chosen from a list of objects and “OK” was pressed. Once the system was created in EViews, the following code was typed into the “Spec” box. After the code was entered, “Estimate” was selected. Then “Two-Stage Least Squares” was chosen from a list of various estimators. Finally “OK” was selected and the system of equations was estimated.

**Model 1**

\[
\text{price} = C(1)+C(2) \times \text{afce}+C(3) \times \text{afcn}+C(4) \times \text{afcs}+C(5) \times \text{altrec}+C(6) \times \text{attend}+C(7) \times \text{cwin}+ \\
C(8) \times \text{nfce}+C(9) \times \text{nfcn}+C(10) \times \text{nfcw}+C(11) \times \text{pwin}+C(12) \times \text{star}+C(13) \times \text{wealth}+ \\
C(15) \times \text{year03}+ (16) \times \text{year04}+C(17) \times \text{pexp}
\]

\[(\text{pricead}+C(6) \times \text{attend})=0 \text{ or } (\text{cap-attend})=0\]

\[
\text{inst afce afcn afcs altrec cwin nfce nfcn nfcw pwin star wealth year03 year04 exp pop}
\]

**Model 2**

\[
\text{price}^1.76 = C(1)+C(2) \times \text{afce}+C(3) \times \text{afcn}+C(4) \times \text{afcs}+C(5) \times \text{altrec}+C(6) \times \text{attend}+C(7) \times \text{cwin}+ \\
C(8) \times \text{nfce}+C(9) \times \text{nfcn}+C(10) \times \text{nfcw}+C(11) \times \text{pwin}+C(12) \times \text{star}+C(13) \times \text{wealth}+ \\
C(15) \times \text{year03}+ (16) \times \text{year04}+C(17) \times \text{pexp}
\]

\[(\text{price}+(C(6)/1.76) \times \text{attend}+(\text{price}^-.76))=0 \text{ or } (\text{cap-attend})=0\]

\[
\text{inst afce afcn afcs altrec cwin nfce nfcn nfcs nfow pwin star wealth year03 year04 pexp pop}
\]

101
Model 3

\[
\begin{align*}
\text{price}^{1.8} &= C(2) \cdot \text{afce} + C(5) \cdot \text{altrec} + C(6) \cdot \text{attend} + C(7) \cdot \text{cwin} + C(8) \cdot \text{nfce} + C(10) \cdot \text{nfcs} + C(11) \cdot \text{nfcw} + C(12) \cdot \text{pwin} + C(14) \cdot \text{wealth} + C(15) \cdot \text{year03} + (16) \cdot \text{year04} \\
\text{(price} + (C(6)/1.76) \cdot \text{attend} \cdot (\text{price}^{-.76})) &= 0 \text{ or (cap-attend)=0} \\
\text{inst afce altrec cwin nfce nfcs nfcw pwin wealth year03 year04 pop}
\end{align*}
\]
SOURCES CONSULTED

Books


Knight, Frank Hyneman. "Risk, Uncertainty and Profit." (1921): 381.


Journal Articles


**Working Papers**


